AB- AND BA-GMRES METHODS FOR X-RAY CT WITH AN UNMATCHED BACK PROJECTOR

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We consider X-ray computed tomography (CT) in the form of a discretization A x = b. This general approach does not assume any specific scanning geometry, and it produces good reconstructions for limited-data and/or limited-angle problems.

The matrix A that represents the forward projector Computation of CT reconstructions also involves a matrix B that represents the so-called back projector and which, ideally, is the transposed of A. In large-scale CT problems, these matrices are too large to store, and we must use software that computes the operations with A and B in a matrix-free fashion. Optimal use of GPUs calls for the use of different discretization methods for the forward and back projectors; hence B is typically different from the transpose of A, and we refer to B as an unmatched back projector.

The consequence is that iterative solvers based on multiplications with A and B solve the "unmatched normal equations" B A x = B b or A B y = b, x = B y. It is natural to use the GMRES algorithm to solve these systems, and our work focuses on the preconditioned AB-GMRES and BA-GMRES methods for solving least squares problems with B as a right and left preconditioner, respectively.

We study the performance and the regularizing effects of the AB- and BA-GMRES methods with and without restart. We also show how to terminate the iterations at the point of semi-convergence before the noise starts to dominate the solution. Our numerical experiments with simulated and measured data demonstrate that AB- and BA-GMRES can be used successfully to solve large-scale CT problems with an unmatched back projector.