BRANCHED ITÔ FORMULA AND NATURAL ITÔ-STRATONOVICH ISOMORPHISM

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Branched rough paths, defined as characters over the Connes-Kreimer Hopf algebra HCK, constitute integration theories that may fail to satisfy the usual integration by parts identity. Using known results on the primitive elements of HCK we can view it as a commutative cofree Hopf algebra (i.e. a commutative B∞-algebra) and thus write an explicit change-of-variable formula for solutions to rough differential equations. This formula, which is realised through an explicit morphism from the Grossman-Larson Hopf algebra to the Hopf algebra of differential operators, restricts to the well-known Itô formula in the very special case of semimartingales. In addition, we establish an isomorphism between HCK and the shuffle algebra over its primitives, which extends Hoffman's exponential for the quasi-shuffle algebra, and can therefore be viewed as a far-reaching generalisation of the usual Itô-Stratonovich correction formula for semimartingales. Indeed, this can be stated as a characterisation of the algebra structure of any commutative B∞-algebra. Compared to previous approaches, this transformation has the key property of being natural in the decorating vector space. We study the one-dimensional case more closely, by introducing the branched analogue of the Kailath-Segall polynomials and Doléans-Dade exponential, and conclude with some examples of branched rough path lifts of a stochastic process which are not quasi-geometric.