# SOME ARITHMETIC ASPECTS OF SINGULARITIES

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Let $f$ be a non-constant reduced polynomial in $n$ variables. The equation $f(x)=0$ defines a hypersurface $X$ in $A^n$. A lot of important information about $X$ or $f$ is found at the singularities of $X$. In fact, many strange phenomena appears when we get closer to singularities. Geometers study the shape, the geometric properties around singularities and the invariants of singularities. Especially, characterizing and classifying singularities are active topics. Besides including geometric information, singularities also storage a lot of deeply arithmetic information. For instance, the solutions of an arbitrary congruence equation of $f$ are often very crowded near the singularities of $X$. Ideally, we may hope a deep relation between the arithmetic information and the geometric information of singularities. This expectation is the content of the (strong) monodromy conjecture. More precisely, given a polynomial $f$ as above such that all coefficients of $f$ are integers, the important information about the number of solutions of the congruence equation $f(x)=0$ modulo $p^m$ is read by the poles of the $p$-adic Igusa local zeta function of $f$. Roughly speaking, the (strong) monodromy conjecture expects that these poles can be expressed in terms of invariants of $X$ which are computed by using tools of algebraic topology or $D$-module theory. In this talk, I will summarize some problems and some known results on the study of some arithmetic aspect of singularities under the philosophy of the (strong) monodromy conjecture.