QUADRATIC CONGRUENCES AND WEYL SUMS

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Given a sequence of real numbers arising from an arithmetic context, how are their fractional parts distributed in the unit interval? There are many interesting instances of this question in number theory.

In a classical context, we may consider a quadratic congruence, varying the moduli and forming a sequence of roots-to-moduli ratios. In 1963, Christopher Hooley showed that if the moduli vary over the natural numbers, then the sequence is uniformly distributed modulo one. The equidistribution modulo one when the moduli take values in the prime numbers is a much more challenging problem. It was first established for negative-discriminant quadratics by Duke-Friedlander-Iwaniec in 1995 and then for positive-discriminant quadratics by Toth in 2012. In both cases, a crucial ingredient is a sufficiently strong estimate for exponential sums of Weyl type associated with the congruence roots.

We plan on giving a general introduction to this circle of equidistribution questions. We will review the beautiful results of Hooley, Duke-Friedlander-Iwaniec, and Toth. We will discuss our improvements on bounds of congruence Weyl sums for positive-discriminant quadratics.