

# Tensors: rank, entropy and entanglement

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## 1. Abstract

We wish to understand when a tensor  $s$  can be transformed into a tensor  $t$  by application of linear maps to its tensor legs (we then say  $s$  restricts to  $t$ ). In the language of restrictions, the rank of a tensor  $t$  is given by the minimal size of a diagonal tensor restricting to  $t$ . The study of rank and restrictions are motivated by algebraic complexity theory, where the rank corresponds to the computational complexity of a bilinear map (e.g. matrix multiplication) which then is viewed as a tensor with three legs.

Interestingly, some important open problems can be formulated in terms of asymptotic properties of restriction, among them the exponent of matrix multiplication. Following the seminal work of Volker Strassen, we will therefore study whether for large  $n$  the  $(n+o(n))$ 'th tensor power of  $s$  can be restricted to the  $n$ 'th tensor power of  $t$ . The information-theoretic flavor of the problem is apparent and was heavily used by Strassen in conjunction with the discovery of algebraic structures (his spectral theorem).

Identifying  $k$ -leg-tensors with states of quantum systems of  $k$  particles allows us to bring tools and ideas from quantum information theory to the table, among them entanglement polytopes and quantum entropy. I will use these to construct a family of functionals - the quantum functionals - that can serve as obstructions to asymptotic restrictions. The functionals are the first of their kind applicable to all tensors, thereby solving a problem by Strassen from 1986.