

ProbParareal: A Probabilistic Numerical Parallel-in-Time Solver for Differential Equations

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We introduce ProbParareal, a probabilistic extension of the Parareal algorithm designed to provide uncertainty quantification for the Parallel-in-Time (PinT) solution of ordinary and partial differential equations (ODEs and PDEs). The method employs (nearest neighbors) Gaussian processes to model the Parareal correction function, enabling the propagation of numerical uncertainty across time and yielding probabilistic forecasts of system evolution. ProbParareal accommodates probabilistic initial conditions and maintains compatibility with classical numerical solvers, which ensures its straightforward integration into existing Parareal frameworks. We conduct a theoretical analysis of the computational complexity and convergence properties of ProbParareal. Numerical experiments on five benchmark ODE systems and a PDE one, including chaotic, stiff, and bifurcation problems, demonstrate the accuracy and robustness of the proposed algorithm. This work bridges a critical gap in the development of probabilistic counterparts to established PinT methods.