Title: Schwarz methods by domain truncation

Abstract: Schwarz methods use a decomposition of the computational domain into subdomains and need to put boundary conditions on the subdomain boundaries. In domain truncation one restricts the unbounded domain to a bounded computational domain and also needs to put boundary conditions on the computational domain boundaries. In both fields there are vast bodies of literature and research is very active and ongoing. It turns out to be fruitful to think of the domain decomposition in Schwarz methods as truncation of the domain onto subdomains. Seminal precursors of this fundamental idea are by Hagstrom et al. (1988), Després (1990) and Lions (1990). The first truly optimal Schwarz method that converges in a finite number of steps was proposed by Nataf (1993) and used precisely transparent boundary conditions as transmission conditions between subdomains. Approximating these transparent boundary conditions for fast convergence of Schwarz methods led to the development of optimized Schwarz methods - a name that has become common for Schwarz methods based on domain truncation. Compared to classical Schwarz methods which use simple Dirichlet transmission conditions and have been successfully used in a wide range of applications, optimized Schwarz methods are much less well understood, mainly due to their more sophisticated transmission conditions.

A key application of Schwarz methods with such sophisticated transmission conditions are time-harmonic wave propagation problems, because classical Schwarz methods simply do not work then. The last decade has brought many new Schwarz methods based on domain truncation. A review from an algorithmic perspective by Gander and Zhang (2019) showed the equivalence of many of these new methods to optimized Schwarz methods. The analysis of optimized Schwarz methods is however lagging behind their algorithmic development. The general abstract Schwarz framework cannot be used for the analysis of these methods, and thus there are many open theoretical questions about their convergence. Like for practical multigrid methods, Fourier analysis has been instrumental for understanding the convergence of optimized Schwarz methods and to tune their transmission conditions. Similar to Local Fourier Mode Analysis in multigrid, the unbounded two subdomain case is used as a model for Fourier analysis of optimized Schwarz methods due to its simplicity. Many aspects of the actual situation, e.g., boundary conditions of the original problem and the number of subdomains, were thus neglected in the unbounded two subdomain analysis. While this gave important insight, new phenomena beyond the unbounded two subdomain models were discovered.

I will give in my presentation a review and precise exploration of convergence behaviors of optimized Schwarz methods based on Fourier analysis taking into account the original boundary conditions, many subdomain decompositions and layered media, both for Helmholtz equations with the good and the bad sign. The transmission conditions used include the lowest order absorbing conditions (Robin), and also more advanced perfectly matched layers (PML), both developed first for domain truncation. I will also show transmission conditions that lead to convergence in a finite number of steps, in the presence of cross points. The work presented here is joint work with Hui Zhang.