Mark Holland: Dichotomy results for eventually always hitting time sta- tistics and almost sure growth of extremes

Abstract: Suppose $(f, \mathcal{X}, \mu)$ is a measure preserving dynamical system and $\varphi : \mathcal{X} \to \mathbb{R}$ a measurable function. Consider the maximum process $M_n := \max\{X_1, \ldots, X_n\}$, where $X_i = \varphi \circ f^{i-1}$ is a time series of observations on the system. Suppose that (u_n) is a non-decreasing sequence of real num- bers, such that $\mu(X_1 > u_n) \to 0$. For certain dynamical systems, we obtain a zero–one measure dichotomy for $\mu(M_n \leq u_n$ i.o.) depending on the sequence u_n. Specific examples are piecewise expanding interval maps including the Gauss map. For the broader class of non-uniformly hyperbolic dynamical systems, we make significant improvements on existing literature for char- acterising the sequences u_n. Our results on the permitted sequences u_n are commensurate with the optimal sequences (and series criteria) obtained by Klass(1985) for i.i.d. processes. Moreover, we also develop new series criteria on the permitted sequences in the case where the i.i.d. theory breaks down.

Our analysis has strong connections to specific problems in eventual always hitting time statistics and extreme value theory. This work is joint with M. Kirsebom, P. Kunde and T. Persson.