OLD AND NEW RESULTS ON THE PLATEAU PROBLEM

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The question whether any closed embedded curve in 3-dimensional space bounds a surface of least area is a classical problem in geometric analysis, going back to Lagrange in the 18th century and the Belgian physicist Joseph Plateau in the 19th century. A first break-through was achieved by Douglas and Rado in 1930/31 in the parametric setting for minimal surfaces of the type of the disc. Building on ideas of Caccioppoli and De Giorgi, Federer-Fleming in the 1960's introduced geometric measure theory as an alternative approach to the problem, which through the work of Allard, Almgren, and - finally - Hardt-Simon culminated in a very general existence result for smoothly embedded minimal surfaces spanning any given curve.
In the talk we quickly review these results and then discuss the more recent approach to the problem via the De Giorgi/Modica-Mortola theory of phase transitions proposed in 1990 by Froehlich-Struwe and revisited by Guaraco-Lynch this year. Moreover, I will present some current work on the 'Plateau flow' as a means to canonically decompose a given surface into a collection of minimal surfaces of bounded topological type spanning the given curve