## FROM ASYMPTOTICS TO BOUNDS: P\'OLYA CONJECTURE, ODES, AND NUMBER THEORY

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There are many situations when asymptotic behaviour of some multi-parameterdependent quantity (when one of the parameters gets large) provides an upper or lower bound for this quantity for all values of parameters. A classical example of this phenomenon is P\'olya's conjecture in spectral geometry stating that the leading term of Weyl's asymptotics of the eigenvalue counting function for the Laplacian in a bounded Euclidean domain is an upper bound (in the Dirichlet case) or a lower bound (in the Neumann case) for the counting function for all values of the parameter. This conjecture is still open in full generality: I'll discuss its recent resolution for disks, balls, sectors and annuli, as well as other similar or related phenomena arising in other subjects, from ODEs to number theory.