Posters for Workshop June 20 -24 2022

Extending the Approximate Component Mode Synthesis Multiscale Discretization to the Helmholtz Equation

Elena Giammatteo, University of Twente, Netherlands

Abstract: In 2010, Hetmaniuk and Lehoucq developed a conforming sepcial finite element method for heterogeneous elliptic partial differential equations based on approximate Component Mode Synthesis (ACMS). The CMS framework exploits the well-known a-orthogonal decomposition of the approximation space into local spaces and a harmonic interface space, where functions in each space are approximated by eigenmodes. The ACMS discretization then approximates the global interface functions by basis functions with local support.

Our work aims to extend the ACMS framework to the Helmholtz equation and to study its convergence properties both numerically and analytically

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Towards matrix-free parallelization of deflation preconditioning for the 3D heterogeneous high-frequency Helmholtz equation

Vandana Dwarka, TU Delft, Netherlands

Abstract: Motivated by the observation that the eigenvalues of the well-known Complex Shifted Laplacian (CSL) preconditioned system shift towards zero as wavenumber increases, deflation techniques were incorporated to accelerate the convergence of preconditioned Krylov subspace methods. The use of higher-order deflation vectors, combined with the CSL preconditioner, can lead to scalable convergence in terms of the number of iterations to reach convergence. Targeting modern large-scale applications, parallelization of the deflation preconditioner is in progress. We start with a matrix-free parallel variant of the CSL preconditioned Krylov subspace method for the 3D heterogeneous Helmholtz equation. The CSL is inverted approximately using one parallel multigrid cycle. The matrix-vector multiplications and preconditioning operators are all implemented in a matrix-free way. Numerical experiments of 3D model problems show that the matrix-free parallel solution method has improved parallel performance and is weakly scalable.

Fast approximation of far-field patterns for polygons

Andrew Gibbs, University College London, UK

Abstract: In many acoustic scattering applications, it is necessary to compute the far-field behaviour induced by a large range of incident angles, to fully understand the scattering properties of the obstacle. In practice this can be computationally expensive, especially at high frequencies. Given a collection of 'canonical' far-field patterns induced by a (frequency independent) number of 'canonical' incident angles, 'embedding formulae' can be used to construct a simple and explicit formula for the far-field pattern, at any incident angle. For example, in the case of scattering by a square, given the far-field patterns induced by eight incident angles, we have an explicit formula for the far-field pattern induced by any incident angle.

Despite being exact in theory, embedding formulae are severely ill-conditioned in practice; when numerical approximations are used for the canonical far-field patterns, embedding formulae suffer from arbitrarily large errors.

In recent work, we have found a way to overcome this ill-conditioning, by adapting the embedding formulae of Biggs 2006, which were derived for polygonal scatterers. This was achieved by exploiting the complex analyticity of the far-field patterns and reformulating the embedding formulae as a contour integral. This representation is shown to be well-conditioned, with respect to the error in the canonical far-field patterns. At the vast majority of observation and incident angles, this contour integral may be computed via residue calculus, making this new class of embedding formulae very efficient at high frequencies.

Analysis of Bernstein-Bézier finite elements for interface problems governed by the time-harmonic maxwell wave equations.

Nawfel Benatia, Cadi Ayyad University, Morocco

Abstract: This work presents a high-order H(curl)-conforming Bernstein-Bézier finite element discretization to accurately solve high-frequency interface problems governed by the time-harmonic maxwell wave equations. The performance of the methods is evaluated using a p-adaptative approach, over a wide range of frequencies, carried out on a set of unstructured triangular mesh grids.

Utilising low frequency dynamics to improve time-stepping capabilities in weather and climate models

Timothy Andrews, University of Exeter, UK

Abstract: Weather and climate PDEs exhibit a considerable time-scale separation, so generally require a very fine temporal discretisation to solve. My research is around enabling a larger time step to be used, whilst retaining sufficient accuracy. Primarily, I consider a new ‘modulation variable’, which evolves on the slower time-scale of the non-linearity. This is constructed through a matrix exponential mapping of the fast, linear, waves. When using a pseudo-spectral approach, analytical expressions can be formed for this operator, in spectral space. Additional phase averaging further reduces local oscillations, leaving a smoother gradient to time-step over. I am analysing the novel introduction of a mean correction term into this mapping. Using this local average of the non-linearity should compensate for phase errors in the modulation variable space. I seek to prove that this will enable greater accuracy with larger time steps in these systems.

Nonlinear inversion using data-driven reduced order models applied to the Helmholtz impedance boundary value problem

Andreas Tataris, Utrecht University, Netherlands

Abstract: Recently, reduced order model techniques have received attention and have been applied to Dirichlet and Neumann inverse boundary value problems. In this poster, we introduce the ROM inversion method applied to the Helmholtz impedance problem. We also discuss the limitations of this approach due to the nature of the governing boundary condition.

An iterative hybrid numerical-asymptotic boundary element method for high-frequency scattering by multiple screens

Oliver Phillips, University of Reading, UK

Abstract: Standard Boundary Element Methods (BEM) for scattering problems, with piecewise polynomial approximation spaces, have a computational cost that grows with frequency. Recent Hybrid Numerical Asymptotic (HNA) BEMs, with enriched approximation spaces consisting of the products of piecewise polynomials with carefully chosen oscillatory functions, have been shown to be effective in overcoming this limitation for a range of problems, focused on single convex scatterers or very specific non-convex or multiple scattering configurations. Here we present a novel HNA BEM approach to the problem of 2D scattering by a pair of screens in an arbitrary configuration, which we anticipate may serve as a building block towards algorithms for general multiple scattering problems with computational cost independent of frequency.

Improving Mass Conservation When applying MINRES to the Stokes Equations with Large Right-Hand Sides

Minhui Zhou, University of Strathclyde, UK

Abstract: We focus on applying the minimum residual method (MinRes) to the linear system arising from finite element discretizations of the Stokes equation system, a fundamental viscous flow model. Due to the imbalance of the linear system and accumulated rounding errors, the accuracy of the linear solver MinRes is affected by the size of the norm of the right-hand side (RHS). This is exacerbated by standard MinRes stopping criteria, that do not appropriately capture the size of RHS. As the magnitude of the forcing term increases, the divergence of the velocity deviates further from zero.

We aim to find a solution that can provide a more accurate solution at an affordable computing cost. This poster shows two potential methods of improvement in the accuracy of MinRes. The first one is to use higher precision within MINRES. The second is to apply a scaled block diagonal preconditioner, which can be equivalently viewed as scaling the Stokes equations. The numerical results showed that the accuracy can be significantly improved by applying these two methods.

Anisotropic orientattion inversion using Stein variational gradient descent

James Ludlam, Katy Tant, Victorita Dolean, and Andrew Curtis

Abstract: In ultrasonic non destructive testing, imaging defects in anisotropic welds presents a significant challenge due to the anisotropic grain structure causing scattering and refraction. Many basic imaging techniques use the total focussing method assuming constant wave speed which leads to unreliable defect detection. If information on the anisotropic orientation of the crystal structure is used to correctly focus a phased array onto a region of interest [1], a significant improvement in flaw detection and accuracy is obtained. Orientation mapping using travel time tomography has been achieved using Markov Chain Monte Carlo (MCMC) Methods [2], however due to the high dimensional parameter space, often consisting of thousands of parameters these methods can take a lot of iterations to form an estimate of the probability distribution. Variational Bayesian Inversion approaches can offer more efficient methods of inversion for these high dimensional parameter spaces as they are posed as an optimisation problem instead of using random sampling. Stein Variational Gradient Descent (SVGD)[3] is one such method which uses the gradient of the probability density function to pick a set of particles to sample the probability density function. Unlike MCMC methods this sample is chosen using optimisation and so can represent the probability density function using a much smaller sample size.
We perform orientation mapping with SVGD using gradients calculated from the fastest ray path between transducers which are obtained using a novel approach to ray tracing through travel time fields. Using these methods we have been able to reconstruct the grain structure in a simple problem consisting of two regions with different anisotropic orientations. Since we obtain an estimate of the probability density function, a measure of uncertainty is obtained for all model parameters.