On the application of spectral deferred corrections to differential-algebraic equations

Lisa Wimmer

Bergische Universität Wuppertal

Originally, the method of spectral deferred corrections (SDC) was developed by A. Dutt et al. (2000) for solving ordinary differential equations (ODEs). SDC is a high- order method of iterative nature where a series of correction equations is be solved to improve the solution in each iteration by using spectral quadrature. ODEs are included in the class of differential algebraic equations (DAEs). DAEs naturally arise in many fields such as power system simulation and chemical engineering. In semi-explicit DAEs the differential part describes the dynamics of the system, where algebraic equations provide conditions to the components and make sure that the dynamics follow certain laws. However, the mixture of differentiation and integration in solving DAEs is a difficult act for numerical solvers. For stiff ODEs, DAEs represent their stiff limit. Thus, explicit solvers cannot find a solution in an acceptable time due to the very small time step size necessary. Implicit solvers, on the other hand, have proven to be successful in solving DAEs and also provide the desired order of accuracy. For semi-explicit DAEs, however, excluding the algebraic part from the integration process can be more efficient. In this talk, the ε-embedding method is presented with which a proper method for solving semi-explicit DAEs can be derived. The idea of the ε-embedding was first proposed by E. Hairer and G. Wanner (2010). They applied it to Runge-Kutta methods to establish a suitable scheme to solve semi-explicit DAEs. We carry over the idea and apply the ε-embedding to establish a suitable SDC method for the numerical solution of semi-explicit DAEs. Theoretical results will be shown, and in numerical experiments the resulting method is compared with other competitive DAE solvers. Further, first steps toward parallelization across the method are also made by applying SDC with different diagonal preconditioners to DAEs. This idea was first proposed by R. Speck (2018) for solving ODEs and PDEs, and in G. Čaklović et al. (2025) an analytical approach is presented to find optimal diagonal coefficients allowing parallelization across the method.