L^q-spectra of self-affine measures:

closed forms, counterexamples, and split binomial sums

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Abstract:

A key aim in fractal geometry is to understand the dimension theory of "irregular" sets and measures. For measures a function known as the L^q-spectrum allows us to quantify this irregularity, whilst also providing information about the geometry of the underlying set.

For nicely behaved measures (such as self-similar measures satisfying an appropriate separation condition) the L^q-spectrum is given by a natural closed form formula, and this is also the case for many self-affine measures. Our main result, which is rather surprising, is that this is not always the case. We do this by providing an explicit family of counterexamples where the expected closed form expression does not generally give the L^q-spectrum. This answers a question posed by Fraser in 2016 in the negative.