The Shaneson–Ranicki splitting is a Bass–Heller–Swan type splitting for Poincaré categories
Markus LAND

I will report on joint work with Calmès, Dotto, Harpaz, Hebestreit, Moi, Nardin, Nikolaus and Steimle. For a Verdier localising invariant F of Poincaré categories, we establish a Bass–Heller–Swan type decomposition for the value of F on the tensor of a Poincaré category with the circle S^1. Applied to Poincaré categories of finitely presented modules over a ring, this gives a formula for the value of F on Laurent polynomials over said ring. For F being algebraic K-theory, we recover the Bass–Heller–Swan decomposition of $K(R[t,t^{-1}])$ (also known as the fundamental theorem of K-theory) including C_2-actions. For F being L-theory, we recover the Shaneson–Ranicki splitting of free L-theory $L(R[t,t^{-1}])$. The decomposition may also be applied to Grothendieck–Witt theory itself, giving an explicit formula for $GW(R[t,t^{-1}])$.

As a consequence of the Shaneson-Ranicki splitting for L-theory, we show that universally decorated L-theory, $L^{\langle -\infty \rangle}$, as appearing in the Farrell–Jones conjecture, is the initial invariant under L-theory (viewed as an invariant of Poincaré categories) which is invariant under idempotent completions. Consequently, universally decorated L-theory is the initial bordism invariant Karoubi-localising invariant of Poincaré categories.