AN EXPLICIT FORMULA FOR PERIOD POLYNOMIALS OF CUSP FORMS

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Given a holomorphic cusp form f, one can associate to it a period polynomial, defined via the Eichler–Shimura correspondence as an integral of f along a geodesic in the upper half-plane. This polynomial satisfies two cocycle conditions arising from the (cuspidal) cohomology of the modular group. Thanks to a result of Gangl, Kaneko, and Zagier, these period polynomials encode all linear relations between formal double zeta values modulo $\zeta(2)$ and modulo lower-depth terms.

After recalling a few key definitions and results in this area, we will introduce a space of bivariate polynomials satisfying commutative analogues of the first two defining relations of the Grothendieck–Teichmüller group. We will explain how this space is linked to the image of the Eichler–Shimura isomorphism, which in turn allows us to give an explicit formula for the period polynomials associated with level 1 cusp forms in any weight. If time permits, we will also discuss some potential consequences of this formula.