BREGMAN RELAXATION OF $\ELL_0$-REGULARIZED CRITERIA WITH GENERAL DATA TERMS

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We propose a new class of exact continuous relaxations of ℓ_0-regularized criteria involving non-quadratic data terms such as the Kullback-Leibler divergence and the logistic regression. Our penalty, named Bregman relaxation (in short, B-rex), is a continuous approximation of the ℓ_0 pseudo-norm defined in terms of suitable Bregman distances. When coupled with a general non-quadratic term, it leads to exact continuous relaxations of the original ℓ_0-regularized problem, in the sense that it does not alter its set of global minimizers and reduces the non-convexity by eliminating certain local minimizers. Both features make the relaxed problem more amenable to be solved efficiently by standard non-convex optimization algorithms, in comparison with greedy algorithms (such as, e.g., matching pursuit and its variants) which often rely on combinatorial approaches whose computational costs explodes as the size of the data increases.
Several numerical results illustrating the geometrical behavior of the proposed relaxation for different choices of the underlying Bregman distance are shown, along with its relation with convex envelopes, as well as its exact relaxation properties in 1D/2D and higher dimensions.