Picard-Fuchs equations and motivic Gamma functions

Matt Kerr

We report on a new development in asymptotic Hodge theory, arising from work of Golyshev-Zagier and Bloch-Vlasenko, and connected to the Gamma Conjectures in Fano/LG-model mirror symmetry. The talk will focus on the Hodge- and period-theoretic aspects.

Given a variation of Hodge structure M on a Zariski open in P^1, the periods of the limiting mixed Hodge structures at the punctures are interesting invariants of M. More generally, one can try to compute these asymptotic invariants for iterated extensions of M by "Tate objects", which may arise for example from normal functions associated to algebraic cycles (or from Feynman integrals).

The main point of the talk will be that (with suitable assumptions on M) these invariants are encoded in an entire function called the motivic Gamma function, which is determined by the Picard-Fuchs operator L underlying M.

The simplest case is when L is hypergeometric: then we get a closed-form expression for this function. In the next simplest class of cases (where deg(L)=2), the leading Taylor coefficient of the motivic Gamma at 1 is given by the special value of a normal function. When M arises from the LG-model of a Fano variety F, such values should be related to certain “Apery constants’’ of F.