

C*-algebras Minicourse

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Lecture 1: We start with a quick introduction to C*-algebras and move on to give a big-picture overview of the classification program for simple C*-algebras and the involved invariants.

Lecture 2: We introduce some intertwining arguments for C*-algebras, building in part on the recommended reading material. We will deduce some consequences from the one-sided intertwining theorem, such as a nonstandard proof of the seminal two-sided Elliott intertwining theorem. If time permits, we discuss how this applies to the classification of AF algebras.

Lecture 3: This is a crash course on the Jiang-Su algebra and the theory of strongly self-absorbing C*-algebras.

Lecture 4: We give an introduction to the theory of finite group actions with the Rokhlin property. After covering some basic examples, we will outline the basic ideas behind the dynamical existence and uniqueness theorems in this context, and indicate how to apply these basic principles for actions on classifiable C*-algebras.

Lecture 5: This lecture is a continuation of the previous one on the classification of Rokhlin actions of finite groups. If time permits, we will discuss how to change the methodological framework in order to classify actions of infinite groups, along with model results that are stated without proof.

Suggested reading: *Classification of Nuclear C*-Algebras* by Rørdam and Størmer, Definitions 1.1.14, 1.1.15 and Proposition 2.3.5 with proof. Optionally sections 2.1 and 2.2.

References for further study:

- Dave Penneys: Lecture notes on higher linear algebra (work in progress). Covers (unitary) tensor categories, fusion categories, and 2-categories. Available here: <https://people.math.osu.edu/penneys.2/8110//Math8110Autumn2023.html>
- Etingof, Gelaki, Nikshych, Ostrik (EGNO): "Tensor categories" (AMS, 2016). Standard reference in the field. Focusses mainly on algebraic aspects. A preprint version of the book can be found on the authors' websites.
- Müger: "Tensor categories: A selective guided tour" (Revista de la Unión Matemática Argentina, 2009). arXiv:0804.3587. Lecture notes for a mini-course on tensor categories, with many references.
- Müger: appendices to Halvorson, "Algebraic quantum field theory" (2006). arXiv:math-ph/0602036. The goal is providing a proof of the Doplicher-Roberts theorem, which says that every symmetric tensor C*-category is the representation category of a compact (super)group, starting from only basic background knowledge.