INDEX THEORY AND SPECTRAL FLOW OF TOEPLITZ OPERATORS

KOEN VAN DEN DUNGEN

Classical Dirac-Schrödinger operators are given by Dirac-type operators on a smooth manifold, together with a potential. I will describe a general notion of Dirac-Schrödinger operators with arbitrary signatures (with or without gradings), which allows us to study index pairings and spectral flow simultaneously. I will first describe a general Callias Theorem, which computes the index (or the spectral flow) of Dirac-Schrödinger operators in terms of index pairings on a compact hypersurface. Associated to each Dirac-Schrödinger operator is also a Toeplitz operator, which is obtained by compressing the potential to the kernel of the Dirac operator. I will then explain how the index or spectral flow of these Toeplitz operators is related to the index or spectral flow of Toeplitz operators on the compact hypersurface. These results generalise various known results from the literature, while presenting them in a common unified framework.