Jack Keeler

Exact solutions for submerged von K´arm´an point vortex streets cotravelling with a wave on a linear shear current

New exact solutions are presented to the problem of steadily-travelling water waves with vorticity wherein a classical von Kármán vortex street cotravels with a wave on a linear shear current. The effects of surface tension and gravity are ignored. The method generalizes an earlier study by Crowdy & Nelson [Phys. Fluids, 22, 096601, (2010)] who found analytical solutions for a single point vortex row cotravelling with a water wave in a shear current. The main theoretical tool is the Schwarz function of the wave and the approach here builds on a novel formulational framework recently set out by Crowdy [J. Fluid Mech., 954, A47, (2022)]. Conformal mapping theory is used to construct Schwarz functions with the requisite properties and to parametrize the associated waveforms. A two-parameter family of solutions is found, requiring the solution of a pair of nonlinear polynomial equations. This system of equations is found have intriguing properties: indeed, it is degenerate, which radically reduces the number of possible solutions, although the space of physically admissible steadily-travelling equilibria is still found to be rich and diverse. For inline vortex streets, where the two vortex rows are aligned vertically, there is in general a single physically admissible solution. On the other hand, for staggered streets, where the two vortex rows are horizontally offset, certain parameter regimes produce multiple solutions and non-uniqueness can occur. An important outcome of the work is that while the classical steady von Kármán point vortex streets cannot exist in an unbounded simple shear current, such equilibria are possible in a shear current beneath a cotravelling wave on a free surface.