A CANONICAL FOLIATION FOR THE BUBBLESHEET SINGULARITIES OF GEOMETRIC FLOWS

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At the core of differential geometry is the notion that the important features of a space should remain invariant under changes of coordinates. Nevertheless, spaces with special structure may admit preferred coordinate systems, highlighting some of its features with particular clarity. Such distinguished parameterisations have often been found by identifying a foliation of the space by submanifolds canonically determined by its geometry. An example is foliations by constant mean curvature (CMC) hypersurfaces, which have been used for instance to parameterise the ends of asymptotically flat manifold, leading to a definition of
center of mass for isolated gravitating systems. They also played a crucial role in the first proof of the stability of Minkowski spacetime, or in foliating geometric ""necks"" to continue geometric flows through neck singularities via surgery.

In the codimension $n \ge 2$ setting, the situation is more complicated. Indeed, where the CMC condition would naturally be replaced by Parallel Mean Curvature (PMC), there are generic geometric obstructions for the establishment of such a foliation. In this work, we introduce a new, pseudodifferential, curvature condition, which we dub ""Quasi-Parallel Mean Curvature"" (QPMC), and find that bubblesheet singularities (the higher codimension counterpart to necks) can be foliated by QPMC embedded spheres. I will present this curvature condition and the construction of the foliation, as well as examples that indicate the necessity of such a condition. Time permitting, I may present some applications to Mean Curvature Flow. This is joint work with Stephen Lynch