Sparsity and tightness of hypergraphs and incidence geometries

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Abstract

A graph $G = (V, E)$ is (a, b)-*sparse* if, for all subsets of m edges covering n vertices, the bound $m \leq an - b$ is satisfied. A graph is (a, b)-*tight* if the graph is sparse and, additionally, $E = aV - b$. The tight $(2, 3)$-graphs are exactly the graphs that are minimally rigid as bar and joint frameworks in the Euclidean plane. The most efficient way to determine if a graph is (a, b)-tight is by using a pebble game algorithm.

An incidence geometry is a combinatorial object that generalises the notion of graph. An incidence geometry with objects of only two types is a hypergraph. The problem of determining the rigidity of certain hypergraphs realised in terms of points and lines in space is used for example in the analysis of the motions admitted by a molecule.

In this talk I will survey what is known about sparsity and tightness of incidence geometries and what these results say about the rigidity of the geometric realisations of these structures. I will also describe how to use group theory to model the situation. New results that will be presented are joint work with Signe Lundqvist, Joannes Vermant and Lars-Daniel Öhman.