Cyclic connectivity of cages

Jozef Rajnik
Comenius University

Abstract

We shall investigate the cyclic connectivity of the cage graphs, where a (k, g)-cage is a smallest k-regular graph with girth g.

A graph G is *cyclically c-edge-connected* if one needs to remove at least c edges to disconnect G into at least two cyclic components. Cyclic connectivity proved to be a profitable structural property of regular graphs, especially cubic ones. The cyclic connectivity of cages has not been studied previously.

We prove that if a (k, g)-cage G has at most $2M(k, g) - O(g^2)$ vertices, where $M(k, g)$ is the Moore bound, then G is cyclically $(k - 2)g$-edge-connected and every cycle-separating $(k - 2)g$-edge-cut in G separates a cycle of length g. In particular, this is true for unknown cages with $(k, g) \in \{(3, 13), (3, 14), (3, 15), (4, 9), (4, 10), (4, 11), (5, 7), (5, 9), (5, 10), (5, 11), (6, 7), (9, 7)\}$ and also the potential missing Moore graph with degree 57 and diameter 2.

Also, we conjecture that all cages have these properties. We base our proof on the extension of the cage problem to the k-regular graphs with girth at least g and s semiedges. We show that the order $|G|$ of such a graph G satisfies a certain quadratic inequality. Consequently, if G contains a cycle, then either $s = (k - 2)g$ and G is the g-cycle, or $|G| \geq M(k, g)/2 - O(g^2)$.

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