Cycle-regularity in vertex-transitive graphs

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Abstract

A graph is said to be *cycle-regular* if for every integer k there exists a constant c_k such that every edge of the graph lies on exactly c_k cycles of length k.

Of course, every edge-transitive graph is cycle-regular, but there is no reason whatsoever to expect that a random cycle-regular graph to be edge-transitive. In fact, one should expect the opposite: almost all cycle-regular graphs are not edge-transitive. It is thus very surprising that under some additional conditions, this reverse can be proved.

For example, as was recently observed by Conder and Zhou, every vertex-transitive cubic graph of girth at most 5 that is cycle-regular is also edge-transitive. Even more recently, this result was extended by Verret and myself to the case of vertex-transitive cubic graphs of girth 6. While this seems as a minuscule improvement, it actually requires some cute arguments ranging over the theory of maps on surfaces, algebra and classical combinatorics. In my talk, I will try to present some of the ideas behind this proof.