On semicubic cages and small graphs of even girth from voltage graphs

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Abstract

A $(3, m; g)$-*semicubic graph* is a graph in which all vertices have degrees either 3 or m and fixed girth g. In this paper, we construct families of semicubic graphs of even girth and small order using two dfferent techniques.

The first technique generalizes a previous construction which glues cubic cages of girth g together at remote vertices (vertices at distance at least $g/2$).

The second technique, the main content of this work, produces bipartite semicubic $(3, m; g)$-graphs with fixed even girth $g = 4t$ or $4t + 2$ using voltage graphs over $\mathbb{Z}_m$. When $g = 4t + 2$, the graphs have two vertices of degree m, while when $g = 4t$ they have exactly three vertices of degree m (the remaining vertices are of degree 3 in both cases).

Specifically, we describe infinite families of semicubic graphs $(3, m; g)$ for $g \in \{6, 8, 10, 12\}$ for infinitely many values of m. The cases $g \in \{6, 8\}$ include the unique 6-cage and the unique 8-cage when $m = 3$.

The families obtained for girth $g \in \{10, 12\}$ include examples with the best known bounds for semicubic graphs $(3, m; g)$.