THE POPULARITY GAP

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Suppose that A is a finite, nonempty subset of a cyclic group of either infinite or prime order. We show that if the difference set $A-A$ is ``not too large'', then there is a nonzero group element with at least as many as $(2+o(1))|A|^2/|A-A|$ representations as a difference of two elements of A; that is, the second largest number of representations is, essentially, twice the average. Here the coefficient 2 is best possible.