Title: Mean field limits for weakly interacting diffusions: mean field limits, phase transitions, multiscale analysis and inference

Abstract: We consider a system of N weakly interacting particles driven by white noise. The mean-field limit of this system is described by the (nonlinear and nonlocal) McKean-Vlasov-Fokker-Planck PDE. We present a detailed analysis of continuous and discontinuous phase transitions for the McKean-Vlasov PDE on the torus. We study the combined diffusive/mean-field limit of systems of weakly interacting diffusions with a periodic interaction potential. We show that, in the presence of phase transitions, the two limits do not commute. We then show the equivalence between uniform propagation of chaos, a uniform-in-N Logarithmic Sobolev inequality, the absence of phase transitions for the mean-field limit, and of Gaussian fluctuations around the McKean-Vlasov PDE. Finally, we develop inference methodologies for estimating parameters in the drift of the McKean SDE using either the stochastic gradient descent algorithm or eigenfunction martingale estimators.