Weak, renormalized, and vanishing-viscosity solutions of the two-dimensional Euler equations

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Let us consider the Euler equations modeling the behavior of an incompressible, homogeneous, inviscid fluid. In the two-dimensional case, the Euler equations can be written in vorticity form as a continuity equation, in which the advecting velocity depends on the vorticity through an integral operator. In my talk, I will introduce several notions of weak solutions for the two-dimensional Euler equations in vorticity form: weak solutions, renormalized solutions, and vanishing-viscosity solutions. Relying on the linear theory for continuity equations with Sobolev velocity field by DiPerna and Lions, I will show that in the subcritical case weak solutions do not exhibit anomalies. In the supercritical case, I will show by means of a duality approach that the same holds for vanishing-viscosity solutions. This has some connections with the two-dimensional theory of turbulence of Kraichnan and Batchelor.