Motion Groupoids

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1. Abstract

The braiding statistics of point particles in 2-dimensional topological phases are given by representations of the braid groups. One approach to the study of generalised particles in topological phases, loop particles in 3-dimensions for example, is to generalise (some of) the several different realisations of the braid group.

In this talk I will construct for each manifold M its motion groupoid Mot_M, whose object class is the power set of M. I will discuss several different, but equivalent, quotients on motions leading to the motion groupoid. In particular that the quotient used in the construction Mot_M can be formulated entirely in terms of a level preserving isotopy relation on the trajectories of objects under flows -- worldlines (e.g. monotonic `tangles').

I will also give a construction of a mapping class groupoid MCG_M associated to a manifold M with the same object class. For each manifold M I will construct a functor $F \colon Mot_M \to MCG_M$, and prove that this is an isomorphism if π_0 and π_1 of the appropriate space of self-homeomorphisms of M is trivial. In particular there is an isomorphism in the physically important case $M=[0,1]^n$ with fixed boundary, for any $n\in\mathbb{N}$.

I will discuss several examples throughout.