Optimized Schwarz methods in time for discrete transport control

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Control problems in which the underlying systems are governed by hyperbolic partial differential equations (PDEs) arise in many applications, such as inverse problems and data assimilation. The numerical solution of such problems is challenging for two different reasons. First, the intensive computations required to simulate hyperbolic phenomena to a high resolution inside the optimization loop means that one must use scalable and efficient solvers that can take advantage of modern parallel architecutres. Second, once the hyperbolic PDE is discretized, numerical dispersion fundamentally changes the behaviour of the problem; thus, iterative algorithms applied to the discrete model behave very differently from what can be predicted based on properties of the continuous problem. In this talk, we present an optimized Schwarz method for solving the transport equation, where we subdivide the time horizon into several sub-intervals and solve the resulting sub-interval control problems in parallel. We prove that for a good choice of parameters, our method converges to the exact solution in a finite number of iterations when applied to the PDE the continuous setting. However, this is no longer true for the discretized PDE; moreover, the best parameter for fast convergence changes depending on whether a relaxation is used, and on whether the iteration is accelerated by a Krylov subspace method. Numerical examples are presented to illustrate this behaviour.

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