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Connections between the Yang-Baxter equation and Thompson's group F

The quantum Yang-Baxter equation is an equation in mathematical physics and it lies in the foundation of the theory of quantum groups. One of the fundamental problems is to find all the solutions of this equation. Drinfeld suggested the study of a particular class of solutions, derived from the so-called set-theoretic solutions. A set-theoretic solution of the Yang-Baxter equation is a pair (X,r), where X is a set and \[r: X \times X \rightarrow X \times X\,,\;\;\; r(x,y)=(\sigma_x(y),\gamma_y(x))\] is a bijective map satisfying $r^{12}r^{23}r^{12}=r^{23}r^{12}r^{23}$, where $r^{12}=r \times Id_X$ and $r^{23}=Id_X\times r$. We define non-degenerate involutive partial solutions as a generalisation of non-degenerate involutive set-theoretical solutions of the quantum Yang-Baxter equation (QYBE). The induced operator is not a classical solution of the QYBE, but a braiding operator as in conformal field theory. We define the structure inverse monoid of a non-degenerate involutive partial solution and prove that if the partial solution is square-free, then it embeds into the restricted product of a commutative inverse monoid and an inverse symmetric monoid. Furthermore, we show that there is a connection between partial solutions and the Thompson's group F. This raises the question of whether there are further connections between partial solutions and Thompson's groups in general.