

# **TRANSMISSION EIGENVALUES AND NON-SCATTERING: SPECTRAL THEORY MEETS FREE BOUNDARY REGULARITY**

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A perplexing question in scattering theory is whether there are incoming time-harmonic waves, at particular frequencies, that are not scattered by a given inhomogeneity. In other words, the inhomogeneity is invisible to probing by such waves. The attempt to answer this question leads to the so-called transmission eigenvalue problem with the wave number as the eigen-parameter. Despite its deceptively simple formulation—two elliptic PDEs in a bounded domain sharing the same Cauchy data—the transmission eigenvalue problem is non-selfadjoint and exhibits a challenging mathematical structure. The non-scattering wave numbers form a subset of the real transmission eigenvalues. Although the existence of infinitely many real transmission eigenvalues is proven for a large class of (not necessarily regular) inhomogeneities, the existence of a non-scattering wave number implies a certain regularity of the inhomogeneity. The proof of the latter employs free boundary regularity techniques. In this talk, we discuss recent developments in the spectral analysis of the transmission eigenvalue problem and non-scattering, and present some interesting related open questions.