Three-dimensional gravity-capillary solitary waves on Beltrami flows

In this talk I consider steady gravity-capillary surface waves ‘riding’ a perfect fluid in
Beltrami flow (a three-dimensional flow with parallel velocity and vorticity fields).

I will first demonstrate how the hydrodynamic problem can be formulated as two equations for two
scalar functions of the horizontal spatial coordinates, namely the elevation of the free surface
and the potential defining the gradient part (in the sense of the Hodge–Weyl decomposition)
of the horizontal component of the tangential fluid velocity there. The formulation is nonlocal, has a variational
structure and generalises the Zakharov–Craig–Sulem formulation for the classical water-wave problem,
reducing to it in the irrotational limit.

Starting from the above formulation, one can derive the Kadomtsev-Petviashvili-I (KP-I) equation
(strong surface tension) or the Davey-Stewartson (DS) system (weak surface tension) for such Beltrami flows
using formal weakly nonlinear theory. These equations have `lump' solutions and thus predict the existence of
fully localised solitary water waves for the full problem. I will show how to rigorously reduce the full problem
to a perturbation of the KP-I or DS equations and thus construct an existence proof for fully localised solitary
waves `riding' Beltrami flows.