ISLAND 6
Dualities and Symmetries in Integrable Systems
(Sabhal Mor Ostaig, Isle of Skye)

Integrable Systems and Mathematical Physics research group
School of Mathematics & Statistics
University of Glasgow

26-30 June 2023

Foreword

It is a great pleasure to count you as one of our participants for this new edition of the ISLAND workshop. We hope that you will appreciate this remote location and that the event will prove to be mathematically stimulating. We remind you that additional information about the workshop can be found on the dedicated website of the *International Centre for Mathematical Sciences* (ICMS):
`https://www.icms.org.uk/DualitiesandSymmetries`

If you are interested in the history of this workshop series, you can have a look at the previous editions:

- ISLAND 1: Integrable Systems: Linear And Nonlinear Dynamics, Islay 1999
`www.maths.gla.ac.uk/island/island1/index.html`
- ISLAND 2: Discrete Systems and Geometry, Arran 2003
`www.maths.gla.ac.uk/island/island2/index.htm`
- ISLAND 3: Algebraic Aspects of Integrable Systems, Islay 2007
`www.maths.gla.ac.uk/island/island3/index.htm`
- (Not)ISLAND 4: Tropical Geometry and Integrable Systems, Glasgow 2011
`www.maths.gla.ac.uk/island/island4/Home.html`
- ISLAND 5: Integrable Systems, Special Functions & Combinatorics, Skye 2019
`www.icms.org.uk/workshops/2019/integrable-systems-special-functions-and-combinatorics`

Scientific committee: Laszlo Feher, Jan Felipe van Diejen, Sasha Veselov.

Local organizers: Christopher Athorne, Maxime Fairon, Misha Feigin, Francesco Giglio, Claire Gilson, Christian Korff, Ian Strachan.

Sponsors: ICMS; Edinburgh Mathematical Society; Glasgow Mathematical Journal Trust.

Contents

Practical Information

The conference location is near Armadale on the Sleat Peninsula on the Isle of Skye (see the map on the next page).

Weather and what to bring. The venue for the workshop is remote and you should make sure you bring everything with you that you need. For example: phone and laptop chargers; plug adaptors for UK plugs and any medication and sunscreen. Possibly the most important thing to bring is an insect repellent. There are few things that blight a visit to Skye more than being bitten by the dreaded midge - this tiny insect can wreak havoc on your skin. There are two well-known repellants Avon Skin-so-Soft and Smidge. We recommend that you bring one of these with you. If you plan to walk in your free time you may want to invest in a net to wear over your head. The weather on Skye changes by the minute: it can be glorious, warm sunshine one moment and then pelting down with rain the next. Please come prepared with waterproofs and layers of warm clothing, bring some good sturdy shoes as well as summer clothing. Have a wonderful time!

Local information. The nearest shop is in Armadale, about 2 miles south of the Gaelic College, the conference venue. The nearest "town" is Broadford to the north and further afield Portree; see the map. There are buses running on Skye, see the **schedule**; they can be unreliable, so have a look at the service website of Stagecoach for updates. Some of the local organisers will be there by car, thus in the case of an emergency please ask us and we will try to give you a lift.


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Free afternoon

Below a list of "local options" for a visit to Armadale Castle (ruin), Museum and Gardens. There are also some local walks, the ones under Option 1 should be easy and provide no difficulty. The ones under Option 3 are a bit further afield and require good hiking shoes, water and windproof clothes (as well as possible protection against midges, i.e. insect repellent). Please consider that there have been warnings concerning the population of midges this summer (see https://www.bbc.com/news/uk-scotland-65939262).

Option 1 – Armadale Castle & Gardens Armadale Castle is about 2 miles south from the Gaelic College: https://www.armadalecastle.com/
Nearby easy walks which can be combined with a visit to the castle and its gardens:
https://www.walkhighlands.co.uk/skye/armadale.shtml
https://www.walkhighlands.co.uk/skye/rubhaphoil.shtml.

Option 2 – Thorabaig Distillery Nearby distillery offering tours at £12 per head (see Thorabaig Distillery website) about 2.4 miles north (50 min walk) of the Gaelic College. The tours are 45mins long. There is also a café.

Option 3 – Local walks Crossing the Sleat Peninsula: start about 4km away from the Gaelic College (transport would be helpful, sturdy hiking shoes required)
https://www.walkhighlands.co.uk/skye/coille-dalavil.shtml.
Southern tip of the peninsula: about 6.3 miles south of the Gaelic College (transport needed, sturdy walking shoes required)
https://www.walkhighlands.co.uk/skye/pointofsleat.shtml;
https://www.walkhighlands.co.uk/skye/sleat.shtml.

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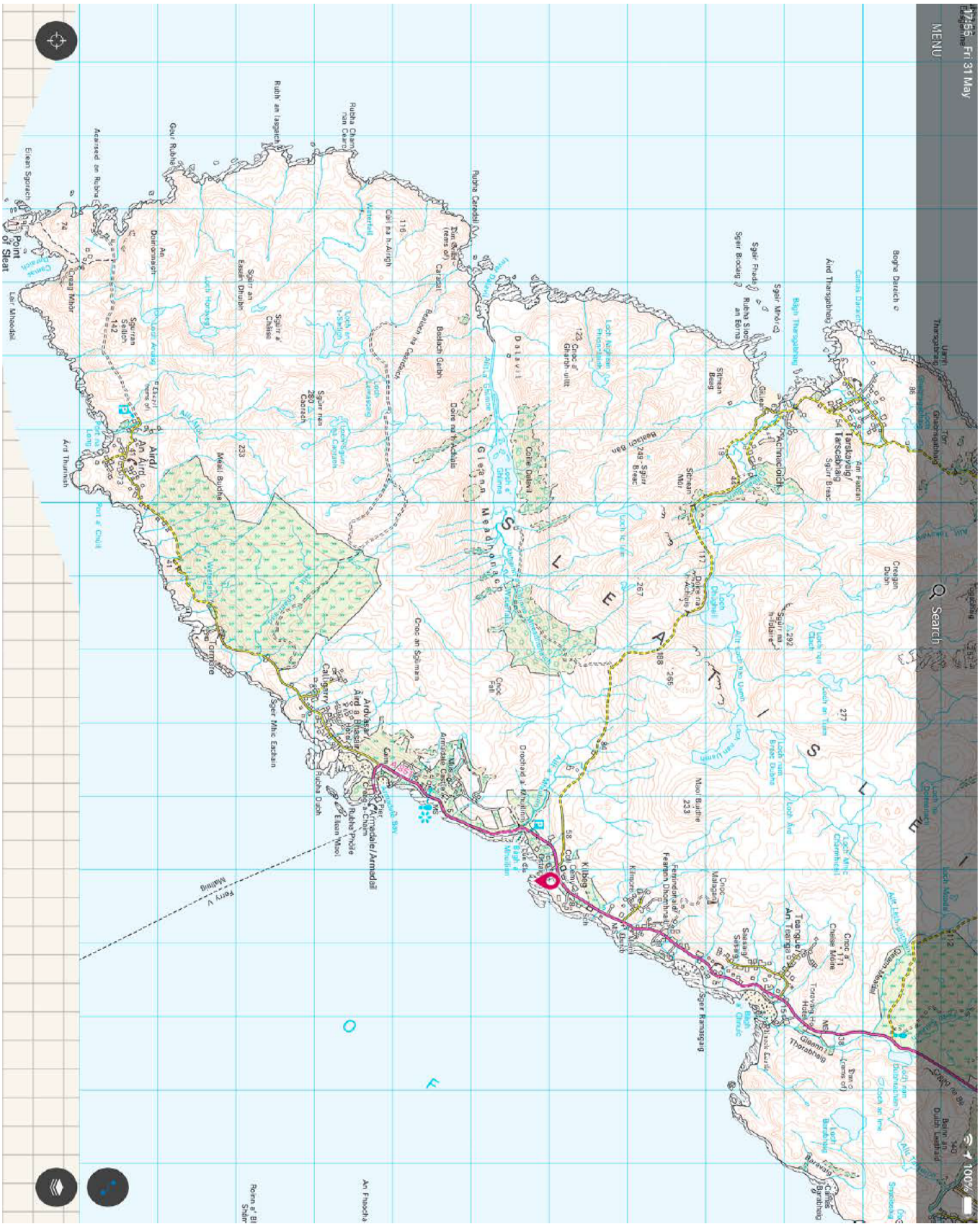
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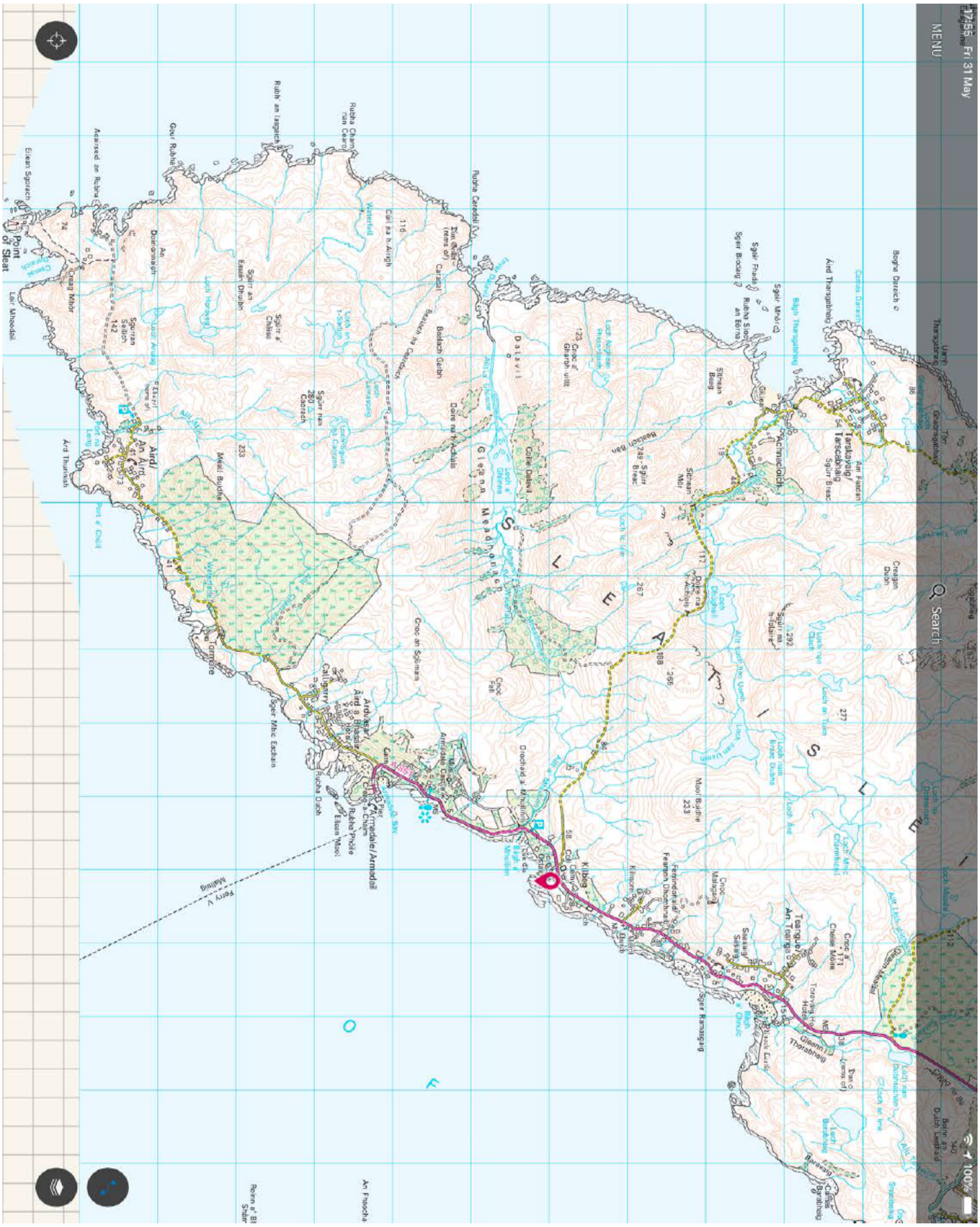
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Calligarry
Armadale/Armadal
Kilbeg
Teangue/ An Teanga
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Schedule

Time	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday
08:00	Arrival	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
08:50		Welcome				
09:00		Veselov	Wang	Joshi	Papamikos	Departure*
09:30		Ferapontov	Novikov	Yang Shi	Adamopoulou	
10:00		Braden	van der Kamp	Dzhamay	Nieszporski	
10:30		Coffee	Coffee	Coffee	Coffee	
11:00		Koroteev	Hone	Sahi	Kels	
11:30		Feher	Kouloukas	Stokman	Roffelsen	
12:00		Young	Maruno	Lamers	Ying Shi	
12:30		Lunch	Lunch	Lunch	Lunch	
13:00						
13:30				Free afternoon		
14:00		Chalykh	Schief		Kassotakis	
14:30		Hallnäs	Willox		Casati	
15:00		Atai	Stokes		Fordy	
15:30		Coffee	Coffee		Coffee	
16:00			Short communications			*Packed lunch to go
16:30						
17:00		Reception				
17:30						
18:00		Dinner	Dinner	Dinner		
18:30			Poster session			
19:00						
19:30					Conference dinner	
20:00	Sandwiches and tea					

Talks

Duality and integrability in topology

26 Jun
9:00

Alexander Veselov
Loughborough University

I will discuss the recently discovered duality between the complex projective spaces and theta divisors as well as the topological aspects of integrability. The talk is based on joint work with V.M. Buchstaber.

Differential equations for modular forms

26 Jun
9:30

Evgeny V. Ferapontov
Loughborough University

I will discuss SL(2,R) and GL(2,R) invariant differential equations for modular forms, as well as plane algebraic curves associated to invariant forms of these equations. Based on joint work with S. Opanasenko.

A New and Old Integrable System

26 Jun
10:00

Harry Braden
University of Edinburgh

By synthesising a number of results in integrable systems and algebraic geometry we will use symmetry to classify and identify spectral curves of BPS monopoles. This will allow us to construct new monopole solutions, something not done for a number of years, and make connection with some old integrable systems. In the process a new integrable system is found.

Opers and Integrability

26 Jun
11:00

Peter Koroteev
UC Berkeley

I will introduce (q)opers as a geometric object which turns out to be extremely effective in working with quantum and classical integrable systems. In particular, opers help in understanding the network of dualities between the aforementioned integrable models.

Poisson reductions of master integrable systems on doubles of compact Lie groups

26 Jun
11:30

Laszlo Feher
University of Szeged and Wigner RCP Budapest

We consider three 'classical doubles' of any semisimple, connected and simply connected compact Lie group G: the cotangent bundle, the Heisenberg double and the internally fused quasi-Poisson double. On each double we identify a pair of 'master integrable systems' and investigate their Poisson reductions. In the simplest cotangent bundle case, the reduction is defined by taking quotient by the cotangent lift of the conjugation action of G on itself, and this naturally generalizes to the other two doubles. In each case, we derive explicit formulas for the reduced Poisson structure and equations of motion, and find that they are associated with well known classical dynamical r-matrices. This yields a unified treatment of a large family of reduced systems, which contains new models as well as well familiar spin Sutherland and Ruijsenaars–Schneider models. It is proved that the reduced systems restricted on a dense open subset of the Poisson quotients are integrable in the degenerate sense. The talk is based on arXiv:2208.03728 complemented by more recent results.

Homotopy Manin triples and higher current algebras

26 Jun
12:00

Charles Young
University of Hertfordshire

The notion of a Manin triple of Lie algebras arises in many contexts in quantum integrable systems and beyond. After recalling the general definition, I will describe one important class of examples involving current algebras, i.e. certain Lie algebras associated to the punctured formal disc in complex dimension one. Studying these examples naturally leads one to recover the ideas of vertex algebras and rational conformal blocks, as I will try to describe.

Now, one would like to generalize all this to higher complex dimensions. (I will sketch one source of motivation, coming from quantum Gaudin models and integrable quantum field theory.) A possible approach to doing so starts with higher current algebras in the sense Faonte, Hennion and Kapranov. I will review the definition, which involves passing from Lie algebras to their differential graded (dg) analogs. In the dg setting, it is natural to relax the definition of a Manin triple, by requiring some statements to hold only up to homotopy. I will describe some recently constructed examples of such homotopy Manin triples, and sketch some applications.

This talk is based on recent work arXiv:2208.06009 and work in progress, joint with Luigi Alfonsi.

Complex crystallographic groups and Seiberg–Witten integrable systems

26 Jun 14:00

Oleg Chalykh
University of Leeds

For any smooth complex variety Y with an action of a finite group W, P. Etingof defines the global Cherednik algebra $H_c = H_c(Y, W)$ and its spherical subalgebra B_c as certain sheaves over Y/W. When Y is an abelian variety, the algebra of global sections $H^0(B_c, Y/W)$ is a polynomial algebra on n generators, as shown by Etingof, Felder, Ma, and Veselov. This defines an integrable system, called a complex crystallographic Calogero–Moser system. In the case of Y being a product of n copies of an elliptic curve E and $W = S_n$, this reproduces the usual elliptic Calogero–Moser system. Recently, together with P. Argyres and Y. Lü, we proposed that many of these integrable systems at the classical level can be interpreted as Seiberg–Witten integrable systems of certain super-symmetric quantum field theories. In this talk I will describe the case of groups W of rank one, which is already interesting.

Kajihara's transformation formula and integrability of deformed Macdonald–Ruijsenaars operators

26 Jun 14:30

Martin Hallnäs
Chalmers University of Technology and the University of Gothenburg

I will explain how specialisations of Kajihara's transformation formula leads to so called (reproducing) kernel identities for generalisations of difference operators of Macdonald–Ruijsenaars as well as Noumi–Sano type. The transformation formula in question is a far-reaching generalisation of an Euler type transformation formula for basic hypergeometric series, whereas the difference operators include the deformed Macdonald–Ruijsenaars operator studied by Sergeev and Veselov.

Eigenfunctions of the van Diejen model generated by gauge- and integral-transforms

26 Jun
15:00

Farrokh Atai
University of Leeds

The quantum van Diejen model is an integrable many-body system defined by a family of mutually commuting analytic difference operators that is known to have hyperoctahedral symmetry in its variables and D_8 (or E_8) Weyl group symmetry in its parameters (under certain constraint).

In this talk, I will start by demonstrating how these symmetries can be realized explicitly in terms of gauge and integral transformations using Ruijsenaars' elliptic Gamma function. Then, I will show that explicit eigenfunctions of the van Diejen model,subject to certain constraints on the model parameters, can be constructed by applying these transformations. In particular, we obtain eigenfunctions given by BC-type elliptic hypergeometric integrals of Selberg type.

The talk is based on joint work with M. Noumi (Rikkyo University).

Quantisations of the Volterra hierarchy

27 Jun
9:00

Jing Ping Wang
University of Kent

In this talk, we'll discuss a recently emerged approach to the problem of quantisation based on the notion of quantisation ideals. We prove that the nonabelian Volterra together with the whole hierarchy of its symmetries admit a deformation quantisation, and that all odd-degree symmetries of the Volterra hierarchy admit also a non-deformation quantisation. The quantisation problem for periodic Volterra hierarchy will also be discussed. In particular, we show that the Volterra system with period 3 admits a bi-quantum structure, which can be regarded as a quantum deformation of its classical bi-Hamiltonian structure. This is a joint work with S. Carpentier and A.V. Mikhailov recently published on Letters in Math. Phys.

Perturbative Symmetry Approach for Nonabelian Differential-Difference Equations

27 Jun
9:30

Vladimir Novikov
Department of Mathematical Sciences, Loughborough University

We propose a new method to tackle the integrability problem for differential-difference equations on free associative algebras (nonabelian differential-difference equations). It enables us to produce necessary integrability conditions, to determine whether a given equation is integrable or not as well as to advance in classification of integrable equations. We define and develop symbolic representation of the non-abelian difference algebra, difference operators and formal series. In order to formulate necessary integrability conditions we introduce a novel quasi-local extension for the algebra of formal series. We apply the developed formalism to solve the classification problem of integrable skew-symmetric quasi-linear nonabelian equations of orders $(-1, 1)$, $(-2, 2)$ and $(-3, 3)$. Some of the obtained equations are new.

Darboux polynomials for ODEs and for mappings, duality for lattice equations, and superintegrable Lotka-Volterra systems

27 Jun
10:00

Peter van der Kamp
La Trobe University

I will discuss the theory of Darboux polynomials, as well as the idea of duality for lattice equations. I show how to find the Darboux polynomials, and hence integrals, for a reduction of a dual to lattice AKP, and how to construct superintegrable n-component $3n - 2$ parameter families of Lotka-Volterra systems.

Heron triangles with two rational medians and discrete integrable systems

27 Jun
11:00

Andrew Hone
University of Kent

Triangles with integer length sides and integer area are known as Heron triangles. Taking rescaling freedom into account, one can apply the same name when all sides and the area are rational numbers. A perfect triangle is a Heron triangle with all three medians being rational, and it is a longstanding conjecture that no such triangle exists. However, despite an assertion by Schubert that even two rational medians are impossible, Buchholz and Rathbun showed that there are infinitely many Heron triangles with two rational medians, an infinite subset of which are associated with rational points on an elliptic curve $E(\mathbb{Q})$ with Mordell-Weil group $\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$, and they observed a connection with a pair of Somos-5 sequences. Here we make the latter connection more precise by providing explicit formulae for the integer side lengths, the two rational medians, and the area in this infinite family

of Heron triangles. The proof uses a combined approach to Somos-5 sequences and associated Quispel-Roberts-Thompson (QRT) maps in the plane, from several different viewpoints: complex analysis, real dynamics, and reduction modulo a prime.

Integrable maps associated with deformations of cluster mutations

27 Jun
11:30

Theodoros Kouloukas
University of Lincoln

We consider parametric deformations of sequences of cluster mutations, in the framework of cluster algebras, which preserve the presymplectic structure defined by the exchange matrix. In the case of non-degenerate exchange matrices parametric symplectic maps are derived. We will study the Liouville integrability of such maps by imposing suitable constraints on the parameters. We also study examples of more general types of mutations, involving fractional linear transformations, related to periodic reductions of integrable lattice equations.

Integrable fully discretizations of multi-component short pulse type equations

27 Jun
12:00

Kenichi Maruno
Waseda University

Solution structure preserving discretizations of integrable systems, i.e., integrable discretizations, have been actively studied in recent years. We have obtained integrable self-adaptive moving mesh schemes of several soliton equations involving hodograph transformations such that the Camassa-Holm equation and the short pulse equation in which mesh intervals are automatically adjusted. In this talk, I will talk about the construction of integrable fully discrete self-adaptive moving mesh schemes for the multi-component short pulse type equations. Multi-soliton solutions are constructed by using pfaffians. This is a joint work with Prof. Yasuhiro Ohta and Prof. Bao-feng Feng.

Non-autonomous integrable deformations of the (heavenly-type) equations governing self-dual Einstein spaces as (symmetry) reductions of the integrable and multi-dimensionally consistent 4+4-dimensional TED equation

27 Jun
14:00

Wolfgang Schief
The University of New South Wales

Recently, a 4+4−dimensional integrable and multi-dimensionally consistent differential equation (TED equation) was introduced and shown how a variety of avatars of the 4-dimensional equation governing self-dual Einstein spaces may be obtained as special cases. This includes Plebanski's first and second heavenly equations and the general heavenly equation. Here, we demonstrate that their integrable non-autonomous deformations, which have been derived in various contexts such as second-order partial differential equations with half-flat (self-dual or anti-self-dual) conformal structure, may likewise be regarded as (symmetry) reductions of the TED equation. Legendre-type transformations turn out to play a key role in the approach.

Testing for integrability using the full-deautonomisation method

27 Jun
14:30

Ralph Willox
The University of Tokyo

The method of full-deautonomisation has proved a strikingly effective way of detecting the dynamical degrees of birational mappings of the plane. It is based on a conjectured link between two a priori unrelated notions: the dynamical degree of the mapping and the evolution of parameters in the mapping that is required for its singularity structure to be preserved under a sufficiently general deautonomisation. In this talk I will introduce the method and explain its advantages over other methods for obtaining the dynamical degree of birational mappings of the plane as well as for some higher order mappings. If time permits I will sketch a proof for the efficacy of the method for some rather large classes of birational mappings of the plane.

Full deautonomisation by singularity confinement as an integrability test: a geometric justification for birational mappings of the plane

27 Jun
15:00

Alexander Stokes
The University of Tokyo

We will present a proof of the conjecture underlying the efficacy of the full-deautonomisation method as an integrability test for a large class of birational mappings of the plane, as introduced in the talk of Ralph Willox. The proof is via the spaces of initial conditions for the deautonomised mappings, and we show that even for non-integrable mappings in this class the surfaces forming these spaces have effective anticanonical divisors and one can define a kind of period map similar to in the theory of discrete Painlevé equations which parametrises these surfaces. This provides a bridge between the evolution of parameters in a deautonomised mapping and the induced dynamics on the Picard lattice which encode the dynamical degree. We will also discuss connections to the theory of rational surfaces associated with root systems of indefinite type and the ways in which our results extend the theory of rational surfaces associated with discrete Painlevé equations.

Dual monodromy surfaces

28 Jun
9:00

Nalini Joshi
The University of Sydney

The Fricke family of cubic surfaces $xyz + x^2 + y^2 + z^2 + a_1x + a_2y + a_3z + b = 0$ is famous in algebraic geometry and in the theory of the sixth Painlevé equation, where it represents the monodromy data arising from a Riemann-Hilbert transform. Monodromy manifolds for the remaining Painlevé equations are also cubic surfaces, which have attracted a great deal of attention. Automorphisms of these surfaces only involve actions on parameters of the corresponding Painlevé equation.

Our recent discoveries show that the situation is quite different for q-discrete Painlevé equations, for which the monodromy data do not necessarily form cubic surfaces. For the q-discrete PVI equation, they are Segre surfaces, defined by 4 quadric polynomials in $\mathbb{P}^6$, while for the q-discrete PIV equation, they form a pencil of cubic surfaces, with the pencil parameter dependent on time in the nonlinear equation. In this talk, we give an overview of these results and describe how they lead to new, symmetric, solutions and Segre surfaces for the classical Painlevé equations.

The results are based on collaborations with Pieter Roffelsen and Marta Mazzocco.

Translations in affine Weyl groups and their applications in discrete integrable systems

28 Jun
9:30

Yang Shi

Flinders University

Recently, we reviewed [1] some properties of the affine Weyl group in the context of their applications to discrete integrable systems such as the discrete Painlevé equations [2]. In particular, a dual representation is used to discuss translational and quasi-translational elements of the Weyl groups. They are found to give rise to the dynamics of various discrete integrable equations.

[1] Y. Shi, Translations in affine Weyl groups and their applications in discrete integrable systems, 63 pages, arXiv:2210.13736.
[2] H. Sakai, Rational surfaces associated with affine root systems and geometry of the Painlevé equations, Commun. Math. Phys. 220(2001), 165-229.

Different Hamiltonians for Painlevé equations and their identification using the geometric approach

28 Jun
10:00

Anton Dzhamay

University of Northern Colorado, USA and BIMSA, China

It is well-known that differential Painlevé equations can be written in a Hamiltonian form. However, a coordinate form of such representation is far from unique — there are many very different Hamiltonians that result in the same differential Painlevé equation. Recognizing a Painlevé equation, for example when it appears in some applied problem, is known as the Painlevé equivalence problem. Here we consider its Hamiltonian version. We describe a systematic procedure for finding changes of coordinates that transform different Hamiltonian representations of a Painlevé equation into some canonical form. Our approach is based on Sakai's geometric theory of Painlevé equations. This is a joint work with Galina Filipuk, Alexander Stokes, and Adam Ligeza.

Metaplectic generalizations of Macdonald polynomials

28 Jun
11:00

Siddhartha Sahi
Rutgers University

Macdonald polynomials are a remarkable family of functions. They are a common generalization of many different families of special functions arising in the representation theory of reductive groups, including spherical functions and Whittaker functions. In turn, Macdonald polynomials can be understood in terms of a certain representation of Cherednik's double affine Hecke algebra (DAHA), acting on polynomial functions on a torus. Whittaker functions admit a natural generalization to the setting of metaplectic covers of reductive p-adic groups, which play a key role in the theory of Weyl group multiple Dirichlet series. It turns out that Macdonald polynomials also admit a corresponding generalization, which can be understood in terms of a representation of the DAHA on the space of quasi-polynomial functions on a torus.

This is joint work with Jasper Stokman and Vidya Venkateswaran.

A new class of vector-valued Macdonald polynomials

28 Jun
11:30

Jasper Stokman
University of Amsterdam

I will introduce a new class of vector-valued nonsymmetric Macdonald polynomials. For type A they generalize the ones introduced by Dunkl and Luque in 2011. I will relate them to the scalar quasi-polynomial extensions of the Macdonald polynomials recently introduced by myself, Sahi and Venkateswaran. I will discuss some of the new insights this is leading to, with focus on those relevant to integrable systems.

Integrable spin-generalisations of the elliptic Ruijsenaars model and long-range spin chains

28 Jun
12:00

Jules Lamers
Insitut de Physique Théorique

The Haldane–Shastry spin chain is a long-range model with Yangian invariance and explicit highest-weight eigenvectors, thanks to a connection to an integrable quantum many-body system. In short: the trigonometric spin-Calogero–Sutherland model (intimately related to affine Hecke algebras) already enjoys these properties (from affine Schur–Weyl duality) and reduces to the Haldane–Shastry chain in the 'freezing' limit. In the first part of my talk I will introduce this model and its q-deformation, which is likewise connected to the trigonometric spin-Ruijsenaars–Macdonald model, and explain how the latter can be seen as a (DAHA) dual of the inhomogeneous Heisenberg XXZ spin chain. In the second part I will present new elliptic generalisations of these models: the elliptic spin-Ruijsenaars model and the q-deformation of the Inozemtsev spin chain. Based on work with V. Pasquier and D. Serban (both IPhT) and R. Klabbers (Humboldt U Berlin).

Solutions to Yang-Baxter type equations related to Darboux transformations

29 Jun 9:00

Georgios Papamikos
University of Essex

I will present several examples of birational maps that satisfy the parametric set theoretical Yang-Baxter equation and its entwining generalisation. The construction makes use of Darboux transformations of well-known soliton equations. I will also present various dynamical properties of the derived maps, such as existence of invariants and associated symplectic or Poisson structures, and I will prove their complete integrability in the Liouville sense.

Extensions of Yang-Baxter maps and their integrability

29 Jun 9:30

Panagiota Adamopoulou
Heriot-Watt University

I will present a non-commutative extension, over Grassmann algebras, of maps which satisfy the set-theoretical entwining Yang-Baxter equation, and which arise from refactorisation problems of supermatrices. I will describe how invariants of these maps can be obtained from the associated monodromy supermatrices. Then, by fixing the order of the Grassmann algebra, I will derive several examples of maps involving commutative variables, and I will discuss some of their integrability properties (Lax representation, invariants).

Compatible maps and discrete integrable systems

29 Jun 10:00

Maciej Nieszporski
University of Warsaw

Difference integrable equations are usually given on vertices of a $\mathbb{Z}^2$ graph. They can be often regarded as a non-unique scalar version of system of equations given on edges of the graph (vector version of the equation). I will focus on justifying the superior role of the system on edges and advantages one gets using idea by Levi and Benguria to treat Bäcklund transformation as increment of a discrete variable. Underlying connections with Yang-Baxter maps will be clarified and the hierarchies that these systems belong to will be presented.

Two-component Yang-Baxter maps

29 Jun
11:00

Andrew Kels
The University of New South Wales

In this talk I will show how two-component Yang-Baxter maps may be obtained from the quasi-classical limit of the Yang-Baxter equation. The leading order quasi-classical expansion gives has the form of a classical Yang-Baxter equation, whose solutions are expressed in terms of special functions, such as the dilogarithm and theta functions. The latter Yang-Baxter equation may be reinterpreted as a functional Yang-Baxter equation for two-component quadrirational Yang-Baxter maps. If a certain skew-symmetry property is satisfied the maps are also reversible. Depending on the type of classical Yang-Baxter equation, one may obtain solutions (Yang-Baxter maps) to both the regular form of the functional Yang-Baxter equation involving an individual Yang-Baxter map, and an entwining form of the functional Yang-Baxter equation involving two different Yang-Baxter maps.

Singularities of Painlevé functions, Heun equations and generalised Hermite polynomials

29 Jun
11:30

Pieter Roffelsen
The University of Sydney

In this talk, I will explain how computing the distributions of singularities of Painlevé functions is equivalent to solving inverse monodromy problems for Heun equations. This equivalence allows for the exact and asymptotic study of singularity distributions through application of Nevanlinna's theory of branched coverings of the Riemann sphere and complex WKB theory to Heun equations. As a main example, I will describe how this framework can be applied to the study of Wronskians of consecutive Hermite polynomials, yielding a proof of a conjecture by Peter Clarkson (2003) as well as special cases of conjectures by Felder et al. (2010).

Matrix-valued Cauchy bi-orthogonal polynomials and a novel noncommutative integrable lattice

29 Jun
12:00

Ying Shi
Zhejiang University of Science and Technology

In this talk, we investigate the relations between the matrix-valued Cauchy bi-orthogonal polynomials and the integrable systems. We give a formal definition for matrix-valued Cauchy bi-orthogonal polynomials and their quasi-determinant expressions. Then, we derive a four-term recurrence relation for the matrix-valued Cauchy bi-orthogonal polynomials and show the coefficients in the recurrence relation could also be written in terms of quasideterminants. Moreover, we introduce proper time flows and hence derive a novel noncommutative integrable lattice. We provide a Lax pair to this novel noncommutative integrable lattice, and verify it through a direct method by quasideterminant identities.

Compatible maps and discrete integrable systems. The non-Abelian case

29 Jun
14:00

Pavlos Kassotakis
University of Warsaw

There is an increasing interest in deriving and extending integrable difference equations to the non-Abelian setting. The non-Abelian setting refers to the dependent variables that participate on the difference equations under consideration, which are considered as elements of a unital associative algebra and in that respect totally non-commutative. In this talk we will present non-Abelian integrable difference equations defined on edges or vertices of a $\mathbb{Z}^2$ graph, along with their associated hierarchies. Finally, the connection of these hierarchies with deformations of the non-Abelian discrete KP hierarchy will be given.

Hamiltonian structures for nonabelian differential-difference systems

29 Jun
14:30

Matteo Casati
Ningbo University

Integrable nonabelian systems are equations of motion in which the field variables take values in a noncommutative algebra, as a matrix one. In a series of papers with Jing Ping Wang (Nonlinearity 2021, CMP 2022) we have investigated the Hamiltonian structure and recursion operators for hierarchies of differential-difference integrable equations, providing a geometrical interpretation that helps to shed some light onto the structure on nonabelian Hamiltonian systems in general. I will present the notions of double multiplicative Poisson vertex algebra and of nonabelian functional polyvector fields, with the latter as the natural language to describe Hamiltonian systems in the familiar geometrical terms. As an application, I will discuss some results towards the classification of scalar Hamiltonian difference structures and present a list of such structures for the nonabelian generalization of well-known integrable systems such as Volterra, Toda and Kaup lattices.

Stationary Flows Revisited

29 Jun
15:00

Allan P. Fordy
University of Leeds

I'll start the talk with a brief review of "What is a stationary flow" and some background on couple KdV hierarchies, the source of all my examples. Stationary flows are integrable, finite dimensional Hamiltonian systems with a Lax representation.

For the Dispersive Water Wave (DWW) hierarchy (2 component cKdV), I derive the Lagrangians for the stationary reductions and present the Hamiltonians for one particular flow (t_2). A tri-Hamiltonian representation for this is given and the structure of the Poisson brackets discussed.

All potentials in the Hamiltonians of the t_2 flows are separable in parabolic coordinates. The Poisson brackets can be generalised to give a tri-Hamiltonian representation for all potentials separable in parabolic coordinates. However, these are no longer stationary reductions and don't have a (known) Lax representation. Even though the Poisson brackets have Casimirs, it is possible to build a "recursion operator" for the "core" of the Poisson tensors, so we can avoid the lengthy procedure of the stationary reduction when building these tensors.

This talk is based on the two papers:

Allan P. Fordy and Qing Huang: Stationary Flows Revisited, *SIGMA*, 19:015 (34 pages), 2023.
Allan P. Fordy and Qing Huang, Stationary Coupled KdV Hierarchies and Related Poisson Structures. (in preparation)

Posters

Symmetric Matrix Ensemble and Hydrodynamic Chains

27 Jun
16:00

Marta Dell'Atti
University of Portsmouth

Random matrix ensembles underpin integrable discrete structures, the Toda lattice for the Hermitian ensemble and the Pfaff lattice for the symmetric ensemble. Integrability is made manifest by the emergence of a hierarchy of commuting Hamiltonian vector fields for infinitely many times, given by the application of the AKS scheme. The continuum limit for the interpolating function of the field variables populating a simplified version of the lattices produces at the leading order a hierarchy of hydrodynamic type, scalar in the first case and chain in the second.

Symmetries between rational and trigonometric WDVV solutions

27 Jun
16:00

Leo Kaminski
University of Glasgow

The WDVV equations are a system of nonlinear PDEs arising, for example, in theory of Frobenius manifolds. Dubrovin introduced Legendre transformations which are special symmetries of the WDVV equations. It was observed by Riley and Strachan in 2007 that Legendre transformations can map rational solutions of WDVV equations to trigonometric ones. We extend this observation to multiparameter families of solutions of type A_n and B_n which were found by Veselov and Chalykh in the rational case. This is joint work with M. Feigin and I. Strachan.

Classical and quantum dynamical r-matrices in 3d gravity

27 Jun
16:00

Juan Carlos Morales Parra
Heriot-Watt University

In this work we will present three different methods to obtain classical dynamical r-matrices associated to the Lie algebras of the local isometry groups regarding 3d gravity. These are: (i) direct solution of the CDYBE, (ii) Inonu-Wigner contraction of the trigonometric-type classical dynamical r-matrix for SO(2,2) and (iii) explicit computation of Alekseev-Meinrenken-type classical dynamical r-matrix. Then we show how these classical dynamical r-matrices could be used to define Poisson and quasi-Poisson structures related to phase spaces appearing in the Chern-Simons formulation of 3d gravity. Finally we discuss some recent results about their corresponding dynamical fusion/exchange matrices and how with them we could construct dynamical versions of possible quantum isometries of the quantum theory.

Integrable systems, Nijenhuis geometry and Lauricella bi-flat structures

27 Jun
16:00

Sara Perletti
Università degli Studi di Milano-Bicocca

Starting from the Frölicher-Nijenhuis bicomplex associated to a tensor of type (1,1) having vanishing Nijenhuis torsion, we introduce a construction of multi-parameter families of regular bi-flat F-manifolds, where the vanishing Nijenhuis torsion tensor is provided by the operator of multiplication by the Euler vector field and where the parameters are as many as the Jordan blocks appearing in its Jordan normal form.

Quantum-classical dualities in integrable systems

27 Jun
16:00

Mikhail Vasilev
University of Birmingham

Quantum-classical duality is an intriguing relation between classical many-body systems of Calogero-Ruijsenaars type and quantum integrable spin chains. This duality has different connections with different areas of mathematics and physics such as quantum cohomology of flag varieties, representation theory of Hecke algebras and etc. In the talk I will give an overview of previously known results about quantum-classical duality and also present some new ones.

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q-Analogue of the degree zero part of a rational Cherednik algebra

27 Jun 16:00

Martin Vrabec
University of Glasgow

Inside the double affine Hecke algebra $\mathbb{H}_{q,\tau}$ of type $\mathfrak{gl}_n$, we define a subalgebra that q-deforms in a natural way the degree zero part of the corresponding rational Cherednik algebra. We prove that our subalgebra is a flat τ-deformation of the semidirect product of the group algebra $\mathbb{C}\mathfrak{S}_n$ of the symmetric group with the image of the Drinfeld–Jimbo quantum group $U_q(\mathfrak{gl}_n)$ under (a q-difference version of) the Jordan–Schwinger representation. We find all the defining relations and an explicit linear basis for our subalgebra, and we fully describe its centre. This is joint work with M. Feigin.

Bispectrality of generalised Calogero–Moser–Sutherland systems

Martin Vrabec
University of Glasgow

We propose a generalised axiomatic definition of a Baker–Akhiezer (BA) function for a (possibly non-reduced) finite collection of vectors with integer multiplicities such that any collinear vectors in the collection are of the form α, 2α. This generalises the definition by Chalykh–Styrkas–Veselov for reduced configurations and Chalykh's work for the root system BC_n.

We consider the generalised trigonometric Calogero–Moser–Sutherland quantum integrable Hamiltonian H associated to the configuration of vectors AG_2, which is a union of the root systems A_2 and G_2. We construct a BA function satisfying our axiomatics for (a positive half of) the system AG_2. We prove that it is an eigenfunction of the Hamiltonian H and that a bispectrality relation holds. We find two corresponding bispectral-dual difference operators of rational Macdonald–Ruijsenaars type in an explicit form. This is an analogue of bispectrality results for root systems due to Chalykh and those due to Chalykh and Feigin for certain deformations of root systems.

A BA function satisfying our definition is expected to exist also for a deformed BC_n configuration (in progress). This is joint work with M. Feigin.

Flat Coordinates of Algebraic Frobenius Manifolds

27 Jun 16:00

Johan Wright
University of Glasgow

We present relations between flat coordinates of the metric and intersection form of D4(a1), a non-polynomial algebraic Frobenius manifold. We also give some information regarding the singularities of the third order derivatives of the almost dual prepotential.

Participants

1. Panagiota Adamopoulou (Heriot-Watt, UK)
2. Farrokh Atai (Leeds, UK)
3. Chris Athorne (Glasgow, UK)
4. Andre Bedell (Glasgow, UK)
5. Adel Ben Moussa (IPhT, FR)
6. Harry Braden (Edinburgh, UK)
7. Matteo Casati (Ningbo, CN)
8. Simone Castellan (Glasgow, UK)
9. Oleg Chalykh (Leeds, UK)
10. Marta Dell'Atti (Portsmouth, UK)
11. Anton Dzhamay (Northern Colorado, USA & BIMSA, CN)
12. Maxime Fairon (Paris-Saclay, FR)
13. Laszlo Feher (Szeged & Budapest, HU)
14. Misha Feigin (Glasgow, UK)
15. Evgeny V. Ferapontov (Loughborough, UK)
16. Allan P. Fordy (Leeds, UK)
17. Francesco Giglio (Glasgow, UK)
18. Claire Gilson (Glasgow, UK)
19. Martin Hallnäs (Chalmers & Gothenburg, SE)
20. Andy Hone (Kent, UK)
21. Nalini Joshi (Sydney AU)
22. Leo Kaminski (Glasgow, UK)
23. Pavlos Kassotakis (Warsaw, PL)
24. Andrew Kels (New South Wales, AU)
25. Christian Korff (Glasgow, UK)
26. Peter Koroteev (Berkeley, US)
27. Theodoros Kouloukas (Lincoln, UK)
28. Jules Lamers (IPhT, FR)

29. Kenichi Maruno (Waseda, JP)
30. Juan Carlos Morales Parra (Heriot-Watt, UK)
31. Maciej Nieszporski (Warsaw, PL)
32. Vladimir Novikov (Loughborough, UK)
33. Georgios Papamikos (Essex, UK)
34. Sara Perletti (Milan, IT)
35. Alessandro Proserpio (Glasgow, UK)
36. Pieter Roffelsen (Sydney, AU)
37. Bradley Ryan (Leeds, UK)
38. Siddhartha Sahi (Rutgers, US)
39. Wolfgang Schief (New South Wales, AU)
40. Ying Shi (Zhejiang, CN)
41. Yang Shi (Flinders, AU)
42. Alexander Stokes (Tokyo, JP)
43. Jasper Stokman (Amsterdam, NL)
44. Ian Strachan (Glasgow, UK)
45. Daniele Valeri (Rome, IT)
46. Peter van der Kamp (La Trobe, AU)
47. Mikhail Vasilev (Birmingham, UK)
48. Alexander Veselov (Loughborough, UK)
49. Claude Viallet (CNRS & Sorbonne, FR)
50. Martin Vrabec (Glasgow, UK)
51. Jing Ping Wang (Kent, UK)
52. Ralph Willox (Tokyo, JP)
53. Johan Wright (Glasgow, UK)
54. Charles Young (Hertfordshire, UK)

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