Topological quantum computation is hyperbolic

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1. Abstract

We show that a topological quantum computer based on the evaluation of a Witten-Reshetikhin-Turaev TQFT invariant of knots can always be arranged so that the knot diagrams with which one computes are diagrams of hyperbolic knots. The diagrams can even be arranged to have additional nice properties, such as being alternating with minimal crossing number. Moreover, the reduction is polynomially uniform in the self-braiding exponent of the coloring object. Various complexity-theoretic hardness results regarding the calculation of quantum invariants of knots follow as corollaries. In particular, we argue that the hyperbolic geometry of knots is unlikely to be useful for topological quantum computation.