Rough differential equations for volatility

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This talk is based on joint work with Ofelia Bonesini, Ioannis Gasteratos and Antoine Jacquier.

We introduce a canonical way of performing the joint lift of a Brownian motion W and a low-regularity adapted stochastic rough path X, extending Diehl-Oberhauser-Riedel (2015). Applying this construction to the case where X is the canonical lift of a one-dimensional fractional Brownian motion (possibly correlated with W) completes the partial rough path of Fukasawa-Takano (2024). We use this to model rough volatility with the versatile toolkit of rough differential equations (RDEs), namely by taking the price and volatility processes to be the solution to a single RDE. We argue that our framework is already interesting when W and X are independent, as correlation between the price and volatility can be introduced in the dynamics. The lead-lag scheme of Flint-Hambly-Lyons (2016) is extended to our fractional setting as an approximation theory for the rough path in the correlated case. Continuity of the solution map transforms this into a numerical scheme for RDEs. We numerically test this framework and use it to calibrate a simple new rough volatility model to market data.