

Quantum graphs

Priyanga Ganesan

1. Abstract

Quantum graphs are an operator space generalization of classical graphs and have close connections to the field of operator algebras and quantum information theory. In this talk, I will provide a brief introduction to the different perspectives to quantum graphs and discuss my research interests in this direction.

Classical and quantum hypergraphs and their homomorphisms

Gage Hoefler

1. Abstract

Following recent efforts to "quantize" areas and objects of study in discrete mathematics, we introduce a notion of a quantum hypergraph by considering spaces of linear operators between finite-dimensional Hilbert spaces. Utilizing the simulation paradigm in information theory, we introduce notions of quantum hypergraph homomorphisms and quantum hypergraph isomorphisms for classical and quantum hypergraphs by considering different no-signalling correlation classes and the hypergraphs the associated information channels induce. We provide examples of separation between classical and quantum hypergraph isomorphism, and discuss their connections to the theory of non-local games.

Border Ranks of Positive and Invariant Tensor Decompositions: Applications to Correlations

Andreas Klingler

1. Abstract

The Schmidt rank of a bipartite state is robust for approximations, as for every state, there is an epsilon-ball of elements having the same or larger rank. It is known that this statement is false for multipartite tensors. In particular, tensors exhibit a gap between their tensor rank and their border rank. The same behavior also applies to tensor network decompositions, for example, tensor networks with a geometry containing a loop. In this talk, we show that gaps between rank and border rank also occur for positive and invariant tensor decompositions. We present examples of nonnegative tensors and multipartite positive semidefinite matrices with a gap for several notions of positive and invariant tensor (network) decompositions. Moreover, we show a correspondence between certain types of quantum correlation scenarios and constraints in positive ranks. This allows showing that certain sets of multipartite probability distributions generated from local measurements on a tensor network state are not closed. Hence, testing the membership of these quantum correlation scenarios is impossible in finite time.

Combinatorial Theory of Matrix Spaces and Its Applications in Quantum Information

Yinan Li

1. Abstract

Duan, Severini, and Winter proposed the study of a specific matrix space as a quantum generalization of graphs, which allows for the formulation and study of a quantum version of Shannon's zero-error capacity problem. In this talk, we further develop the combinatorial theory of matrix spaces through the lens of graph theory. Initially, we introduce basic correspondences between matrix space properties and graph-theoretical properties, such as nilpotency versus acyclicity, irreducibility versus connectivity, and dimension expansion versus vertex expansion. Subsequently, we demonstrate how these correspondences can be enhanced to the so-called inherited correspondences, which lead to extremal problems for matrix spaces and have applications in invariant theory and noncommutative algebra. Finally, we discuss applications in quantum information processing and provide examples of graph-theoretic properties that are no longer valid in the matrix space setting.