Boundary algebras and local topological order

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1. Abstract

Bravyi, Hastings, and Michalakis introduced certain topological quantum order (TQO) axioms to ensure gap stability of a commuting projector local Hamiltonian and stabilize the ground state space with respect to local operators in a quantum spin system. In joint work with Corey Jones, Pieter Naaijkens, and Daniel Wallick (arXiv:2307.12552), we study nets of finite dimensional C*-algebras on a 2D $\mathbb{Z}^2$ lattice equipped with a net of projections as an abstract version of a quantum spin system equipped with a local Hamiltonian. We introduce a set of local topological order (LTO) axioms which imply the TQO conditions of Bravyi-Hastings-Michalakis in the frustration free commuting projector setting, and we show our LTO axioms are satisfied by known 2D examples, including Kitaev's toric code and Levin-Wen string net models associated to unitary fusion categories (UFCs).

From the LTO axioms, we can produce a canonical net of algebras on a codimension 1 $\mathbbm{Z}$ sublattice which we call the net of boundary algebras. We get a canonical state on the boundary net, and we calculate this canonical state for both the toric code and Levin-Wen string net models. Surprisingly, for the Levin-Wen model, this state is a trace on the boundary net exactly when the UFC is pointed, i.e., all quantum dimensions are equal to 1. Moreover, the boundary net for Levin-Wen is isomorphic to a fusion categorical net arising directly from the UFC. For these latter nets, Corey Jones' category of DHR bimodules recovers the Drinfeld center, leading to a bulk-boundary correspondence where the bulk topological order is described by representations of the boundary net.