Bayesian Variational Full Waveform Inversion to Image the Earth's Subsurface

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Seismic Tomography is a method to image the Earth's subsurface using acoustic and elastic waves. In order to better interpret the resulting images it is important to assess imaging uncertainties, but this is hard to achieve. Monte Carlo random sampling methods are often applied for this purpose but the 'curse of dimensionality' makes them computationally intractable for high-dimensional parameter spaces. To extend uncertainty analysis to larger systems, variational inference methods developed recently in the machine learning community are introduced to seismic tomography. In contrast to random sampling, variational methods solve an optimization problem yet still provide probabilistic results. Variational inference is applied to solve two types of tomographic problems: full waveform inversion (FWI), and time-dependent (known as 4D) FWI. Three different variational methods are tested: automatic differential variational inference (ADVI) and both deterministic and stochastic versions of Stein variational gradient descent (SVGD). ADVI provides a robust mean velocity model but biased uncertainties, whereas deterministic SVGD produces an accurate match to the results of Monte Carlo analysis, but at a fraction of the computational cost. SVGD is significantly easier to parallelize, and for very large problems can be run in minibatch mode without incurring probabilistic biases. Stochastic SVGD is shown to be the only method that may be capable of providing useable uncertainties for 3D FWI problems. The 3D solution is verified independently using a recently developed extension to ADVI. Variational methods have been extended to time-dependent monitoring problems of the type expected to be encountered in CO_2 or Hydrogen storage applications. Similar methods can be used to extend probabilistic analysis to other nonlinear Geophysical inverse problems, and to higher dimensional tomographic systems than is currently thought possible.
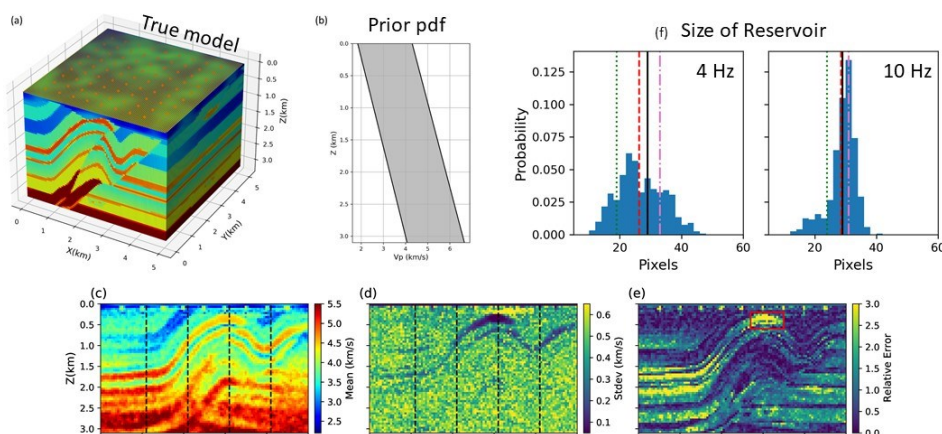
Figure: (a) 3D synthetic model. (b) Uniform prior probability density function. (c),(d),(e): Respectively the mean, standard deviation and relative error (difference between true and mean models, divided by the standard deviation) across central vertical slice in x-z plane. (f) Results of an interrogation problem to answer the question, "*How large is this storage reservoir?*" given probabilistic FWI results, using two different wavelet central frequencies: black line indicates the correct value, red line is the statistically optimal estimate.