

Title: Scalable solvers for the Helmholtz problem

Abstract: For more than 15 years, the industry has relied on using the complex shifted Laplacian (CSL) preconditioner to solve discretized Helmholtz problems. While this works efficiently for medium sized wave numbers, the number of iterations are still too high for practical applications and the problem sized become too large when we move to modern high-frequency problems.

A natural choice for scalable solvers would be to use multilevel methods. In this talk we will focus on both multilevel deflation and multigrid for these highly indefinite problems.

Getting multigrid solvers to work for the Helmholtz equation has been an open problem in applied mathematics for years. We present one of the first stand-alone multigrid solvers for the Helmholtz equation. We use standard smoothing techniques and do not require any restrictions on the number of grid points per wavelength on the coarse-grid. As a result we are able to obtain a full V- and W-cycle algorithm. The key features of the algorithm are the use of higher-order intergrid transfer operators, and a complex shift in the Jacobi smoothing operator.

Next we compare the performance and differences with a multilevel deflation approach. Here, we extend a two-level deflation method to a multilevel deflation method. By using similar higher-order deflation vectors, the near-zero eigenvalues of the the coarser grid operators remain aligned with the fine-grid operator keeping the spectrum of the preconditioned system fixed away from the origin.

Both proposed algorithms provide an important step towards the perpetuating branch of research in finding scalable solvers for wave propagation problems. We illustrate them in this talk by exploring both constant and variable heterogeneous model problems and addressing future challenges.