Dynamical systems for arithmetic schemes

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For any normal scheme X of finite type over spec Z we construct a continuous time dynamical system whose periodic orbits come in compact packets that are in bijection with the closed points of X.
All periodic orbits in a given packet have the same length equal to the logarithm of the order of the residue field of the corresponding closed point. For spec Z itself we get a dynamical system whose periodic orbits are closely related to the prime numbers. The construction uses new ringed spaces obtained by sheafifying rational Witt vector rings. In the zero-dimensional case there is a close relation to work of Kucharczyk and Scholze who realized Galois groups of fields containing the maximal cyclotomic extension of the rational number field as étale fundamental groups of ordinary topological spaces. A p-adic variant of our construction turns out to be closely related to the Fargues-Fontaine curve of p-adic Hodge theory.