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Categories like Hilbert spaces

"The category of Hilbert spaces and bounded linear functions forms the mathematical basis for quantum theory. But why? What physical principles enforce this mathematical structure? This category is also where C^* -algebra theory lives, as it's the universal C^* -category. But why? What properties does it have that accommodate this mathematical structure? As a first answer, this talk provides axioms that guarantee a category is equivalent to that of Hilbert spaces and bounded linear functions. The axioms are purely categorical and do not presuppose any analytical structure such as complex numbers, continuity, dimension, convexity, probabilities, etc. We will also discuss variations, such as linear contractions, finite-dimensional Hilbert spaces, and Hilbert C^* -modules.

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