DIRAC EIGENVALUES, THE HYPERSPHERICAL RADIUS AND APPLICATIONS

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For closed connected Riemannian spin manifolds an upper estimate of the smallest eigenvalue of the Dirac operator in terms of the hyperspherical radius is proved. When combined with known lower Dirac eigenvalue estimates, this has a number of geometric consequences.
They include known results like Llarull's scalar curvature rigidity of the standard metric on the sphere, Geroch's conjecture on the impossibility of positive scalar curvature on tori and a mean curvature estimate for spin fill-ins with nonnegative scalar curvature due to Gromov, including its rigidity statement recently proved by Cecchini, Hirsch and Zeidler.
New applications provide a comparison of the hyperspherical radius with the Yamabe constant and improved estimates of the hyperspherical radius for various types of special geometries.