

The polynomial Szemerédi theorem and beyond

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Abstract:

The polynomial Szemerédi theorem of Bergelson and Leibman is a central result at the interface between ergodic theory and additive combinatorics, extending earlier results of Szemerédi and Furstenberg on arithmetic progressions. It states that each dense subset of integers contains certain polynomial configurations. The theorem follows from an ergodic theoretic result on the convergence of multiple ergodic averages with polynomial iterates. The limiting behaviour of such averages has been an object of intensive study by ergodic theorists and additive combinatorists alike. In this talk, I will discuss new results in this direction. Specifically, I will characterise the limits of multiple ergodic averages related to certain families of polynomial configurations for which little has been known previously. While doing so, I will highlight an interesting connection between the form that these limits take and the algebraic relations satisfied by the polynomials. I will also discuss some consequences of this result, in particular a multiple recurrence result that extends the classical recurrence theorem of Khintchine.