

Interacting particle systems arise in diverse contexts, including to model magnetism in statistical mechanics, opinion and disease dynamics in society, competition between species, and neural networks in machine learning. For large populations evolving over a long period of time, the random fluctuations in these systems average out and their evolution can be described deterministically. However, the particle process will exhibit large deviations away from its mean. These events though rare can have substantial effects---such as a large concentration of energy or the appearance of a vacuum---and they are therefore important to understand and simulate. The purpose of this talk is to introduce a continuum model that replicates the far-from-equilibrium behavior in interacting particle systems. While such approximations have long been used to model out-of-equilibrium behavior in fluctuating hydrodynamics, their use has lacked a rigorous mathematical justification due to the supercriticality and degeneracy of the associated stochastic equations.

We will first introduce a general probabilistic framework for describing fluctuations in random systems. We will then explain how these ideas are applied to interacting particle systems, and how such considerations provide a formal link to certain stochastic PDE. We have developed a robust existence and uniqueness theory for such equations, and shown that along appropriate scaling limits the solutions correctly describe the random fluctuations and rare events in particle systems. These results make rigorous the longstanding formal connection above, and provide new techniques to understand and model particle processes.