

Hilbert's Hotel and other encounters with infinity

How do we measure infinity? How do we compare infinite sets? If someone tells you that “it is equally likely for a prime number to have residue $+1$ as it is likely for it to have residue -1 modulo 4 ”—forgetting any proof of this claim for the moment—in how many ways might one formulate (correct) rigorous mathematical statements that are in accord with that claim?

I will elaborate on such questions, and on heuristics that sometimes guide us to make plausible conjectures about them, in hopes of launching a general discussion.