Michael Atiyah and Physics

30th Anniversary Meeting of the European Mathematical Society

Bernd Schroers, Heriot-Watt University ICMS, 31 March 2022 It is indeed fortunate that the EMS will be launched at a unique historical moment when so many barriers are breaking down...

The political changes now taking place in Europe have an immense potential not least in the cultural and scientific area. The EMS is coming into existence at just the right moment ... to see that mathematics plays its rightful role in the new Europe that we all hope is now being formed.

Atiyah, M. (1991). The European Mathematical Society

Memories

Hosting Roger Penrose, July 2017





In discussion with Andrew Ranicki, January 2016

Heidelberg Laureate Forum 2015



The Scottish Independence Referendum 2014

Atiyah adds voice to Yes campaign over Trident



Sir Michael Francis Atiyah says he will vote Yes in September JAMES GLOSSOP/THE TIMES



Sept 2012: Nick Manton's 60th Birthday



Summer 2011: SFC support for AIMS



Physics heroes

James Clerk Maxwell



$$dF = 0, \quad d * F = j, \quad F \in \Omega^2(M)$$

gauge theory and Lorentz symmetry





Albert Einstein

Physics as (pseudo-) Riemannian geometry

$$\operatorname{Ric}_g - \frac{1}{2}R_g g + \Lambda g = 8\pi GT$$

Unified field theory: particles as `solitons' in non-linear partial differential equations



Paul Dirac

$$D \!\!\!/ \psi = m \psi$$

Spin and differential equations from geometry



Large numbers in physics





Physics collaborators

Ed Witten



Gauge theory: Chern-Simons theory and knot invariants Yang-Mills theory and 4-manifold invariants

String theory and M-theory

Nick Manton

Geodesic approximation to monopole dynamics

Skyrmions from Instantons

Geometric models of matter



The inadvertent physicist

My knowledge of physics was very slim, despite having attended a course on quantum mechanics by Dirac himself.

Michael Atiyah Collected Works, Vol 4 Index Theory 2 (1973-84) My knowledge of physics was very slim, despite having attended a course on quantum mechanics by Dirac himself.

My analytical background was very weak and I remember having to be instructed in the significance of the Fourier transform.

> Michael Atiyah Collected Works, Vol 4 Index Theory 2 (1973-84)

Atiyah-Singer Index Theorem

Atiyah OSinger TOPOLOGICAL VINEYARDS Ind $(\partial_E) = \int_M \hat{A}(M) \cdot ch(E)$ SPECIAL 80TH BIRTHDAY RESERVE VINTAGE 1929

I would have been extremely surprised if I had been told that this work would in due course become important in theoretical physics

> Michael Atiyah Collected Works, Vol 4 Index Theory 2 (1963-84)

I am really quite struck by the way most of the work which Singer and I did in the 60s and 70s has become relevant to physics

> Michael Atiyah Collected Works, Vol 5 Gauge theory (1977-85)

ADHMN Instanton construction

$$*F = \pm F$$

The first time abstract modern mathematics has been of use' Alexander Polyakov cited in Atiyah, Collected Works Vol 4

Monopole moduli spaces



From 1977 .. my interest moved in the direction of gauge theories and the interaction between geometry and physics. I had for many years had a mild interest in theoretical physics

> Michael Atiyah Collected Works, Vol 5 Gauge Theory (1977-85)

The intentional physicist

The philosophy of Dirac...

the experimental data in mathematical terms. The theoretical worker in the future will therefore have to proceed in a more indirect way. The most powerful method of advance that can be suggested at present is to employ all the resources of pure mathematics in attempts to perfect and generalise the mathematical formalism that forms the existing basis of theoretical physics, and *after* each success in this direction, to try to interpret the new mathematical features in terms of physical entities (by a process like Eddington's Principle of Identification).

PMA Dirac, Proc. R. Soc. Lond. A 1931

...and Einstein:

A complete field theory knows only fields and not the concepts of particle and motion. For these must not exist independently of the field but are to be treated as part of it. On the basis of the description of a particle without singularity one has the possibility of a logically more satisfactory treatment of the combined problem: The problem of the field and that of motion coincide.

A Einstein and N Rosen, Physical Review 1935

`It would be quite enough to understand the electron'

A Einstein (according to correspondence between Michael Atiyah and Jeroen van Dongen)

Skyrmions



Figure 1: Skyrmions for $1 \le B \le 8$, with m = 0. A surface of constant baryon density is shown, together with the baryon number and symmetry group. A surface of constant energy density looks similar.

N Manton and Paul Sutcliffe, Topological Solitons, CUP 2004

Geometry and Kinematics of Two Skyrmions

M.F. Atiyah¹ and N.S. Manton²

¹ Trinity College, Cambridge CB2 1TQ, UK

² Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge CB3 9EW, UK

Received June 15, 1992; in revised form October 13, 1992

Abstract. In Skyrme's soliton model of baryons, a single Skyrmion has six degrees of freedom, so it is expected that two-Skyrmion dynamics at modest energies can be modelled by motion on a 12-dimensional space of Skyrme fields. A candidate for this space is generated by the gradient flow of the potential energy function, descending from the unstable, baryon number two, hedgehog solutions of the Skyrme field equation. An apparently very similar space is obtained by restricting the gradient flow to the Skyrme fields derived from SU(2) Yang-Mills instantons of charge two.

M Atiyah and N Manton, CMP 1992

Quantum states of Skyrmions



Manton, Leese, BJS, Attractive channel Skyrmions and the deuteron, Nucl. Phys. B442 (1995) 228-267

Configuration space conjecture

Michael Berry: Find map:

 $f_n: \mathcal{C}_n(\mathbb{R}^3) \to U(n)/T^n$

which intertwines action of permutation and rotation group

define
$$t_{ij} = \frac{x_i - x_j}{|x_i - x_j|},$$
Michael Atiyah:
 $x_1, \dots, x_n \in \mathbb{R}^3, \quad i \neq j$ $p_j(z) = \prod_{i \neq j} (z - t_{ij})$ Conjecture: $f_n : (x_1, \dots, x_n) \mapsto (p_1, \dots, p_n)$ solves Michael Berry's
problem!

The geometry of point particles

 BY MICHAEL ATIYAH¹ AND PAUL SUTCLIFFE²
¹Department of Mathematics and Statistics, University of Edinburgh, King's Buildings, Edinburgh EH9 3JZ, UK (atiyah@maths.ed.ac.uk)
²Institute of Mathematics, University of Kent at Canterbury, Canterbury CT2 7NZ, UK (p.m.sutcliffe@ukc.ac.uk)

Received 23 May 2001; accepted 3 September 2001; published online 3 April 2002

Difference-differential equations

A Shifted View of Fundamental Physics

Michael Atiyah and Gregory W. Moore (2010)

ABSTRACT: We speculate on the role of relativistic versions of delayed differential equations in fundamental physics. Relativistic invariance implies that we must consider both advanced and retarded terms in the equations, so we refer to them as shifted equations. The shifted Dirac equation has some novel properties. A tentative formulation of shifted Einstein-Maxwell equations naturally incorporates a small but nonzero cosmological constant.

$$\dot{x}(t) + kx(t-r) = 0$$
 $\{i\hbar D - mc + ik\exp(-rD)\}\psi = 0$

Geometric models of matter



- 4-manifolds as static particles
- dual Kaluza-Klein picture
- quantum numbers from topology
- spin states via Dirac operator





Recurring themes

- The importance of beauty
- Trust great minds!
- Quantum mechanics flawed (linear!) and not fundamental. Bohm's interpretation better!
- Gauge theory not fundamental: too much freedom
- String theory important but not central
- Central role of geometry and non-linearity
- 4-manifolds are special
- Division algebras as natural symmetries
- Dirac operator as bridge between geometry and QM
- Intuition beats data...

The teacher

MICHAEL F. ATIYAH Topological quantum field theory

Publications mathématiques de l'I.H.É.S., tome 68 (1988), p. 175-186



Conclusion

Michael felt a strong kinship with Hermann Weyl



Both made profound contributions to physics - but perhaps did so most successfully when acting inadvertently.

However, Michael's instinct in physics may still turn out to be right!