

Nonlinear diffusion equations can be rigorously derived from stochastic interacting many-particle systems in the mean-field limit, as proved by Jourdain/Méléard and Oelschläger. We extend the ansatz of Oelschläger for moderately interacting many-particle systems to the case of multiple species, leading to cross-diffusion systems for the limiting probability density functions. First, if the drift depends on the interactions, cross-diffusion systems with quadratic nonlinearity are rigorously derived, possessing a double entropy structure. Second, if the diffusion coefficient of the particle system depends on the interactions, more general cross-diffusion systems are obtained, including the well-known Shigesada-Kawasaki-Teramoto population system. We prove not only the convergence of the stochastic processes but also provide an estimate for the mean-squared error of the stochastic processes. Finally, we present a random-batch method for the efficient numerical solution of the interacting particle systems.