Title: Stable approximation of Helmholtz solutions by evanescent plane waves

Abstract: Solutions of the Helmholtz equation are known to be well approximated by superpositions of propagative plane waves. This observation is the foundation of successful Trefftz methods. However, when too many plane waves are used, the computation of the expansion is known to be numerically unstable. This effect is due to the presence of exponentially large coefficients in the expansion and can severely limit the efficiency of the approach.

We show that the Helmholtz solutions on a disk can be exactly represented by a continuous superposition of evanescent plane waves, generalizing the classical Herglotz representation. The density in this representation is uniformly bounded in a weighted Lebesgue norm, hence overcoming the instability observed with propagative plane waves. This allows to construct suitable finite-dimensional sets of evanescent plane waves using sampling strategies in a parametric domain. Provided one uses sufficient oversampling and regularization, the resulting approximations are shown to be both controllably accurate and numerically stable, as supported by numerical evidence.

This is joint work with E. Parolin (Pavia) and D. Huybrechs (KU Leuven).