

Monodromy of monodromy manifolds and symmetries of Sakai surfaces

Alexander Stokes

Painlevé equations are associated to complex algebraic surfaces in two different ways, which are related by biholomorphism under different instances of the Riemann-Hilbert correspondence. On the ‘left-hand side’ are Sakai surfaces, which provide Okamoto’s initial value spaces, and on the ‘right-hand side’ are monodromy manifolds coming from associated linear problems. Symmetries of Sakai surfaces form extended affine Weyl groups and play a crucial role as Bäcklund transformation symmetries of the Painlevé equations. However, under the Riemann-Hilbert correspondence these become trivial on monodromy manifolds.

In this talk we announce results showing that there is still a shadow of the extended affine Weyl group symmetry on the monodromy manifold side, but this lies outside of the class of automorphisms of varieties. This shadow is formed by the monodromy group of the monodromy manifold itself, which we show matches exactly with the underlying finite Weyl group from the symmetries of the corresponding Sakai surface.

Based on joint work with Pieter Roffelsen.